Piecewise Union Blend with Adjustable Primitive's Subsequent Blend Range Parameters in Constructive Geometry

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***Abstract*—In constructive geometry, implicit blends with blend range parameters are important in constructing a complex surface because they limit the blend surface within specified regions and deform primitive locally after blending. However, because they behave like *Max* or *Min* blend in non-blending regions, their primitives always have similar sizes of subsequent blend surfaces when used as a primitive in sequential blends. To solve this problem, this paper proposes union blends that provide each primitive an additional parameter to adjust a primitive's subsequent blending range in later blends; thus, primitives are able to individually adjust the sizes of their subsequent blends by varying the values of the additional parameters. In addition, a generalized method is proposed to develop the proposed union blends with C^1 continuity from an existing blend.**

***Index Terms*—implicit surface, implicit blends, constructive geometry**

I. Introduction

In Constructive geometry of implicit surfaces, primitive implicit surfaces are defined as 1 level surface of a defining function. Defining functions determine the shapes of primitives, which can be found in [1]-[6]. A complex surface is then constructed easily from primitive surfaces, such as planes, ellipsoids, skeletal primitives, etc. by implicit blends. A blend connects and joins primitives with transitions generated automatically to smooth out unwanted sharp edges and creases. Existing blends in Constructive geometry in the literature are reviewed as follows:

- *Max* and *Min* functions offer pure union and intersection [7]. Because they are C^0 continuous only, they always generate non-smooth surfaces.
- Super-ellipsoidal blends [7] offered union and intersection blends of high-order continuity by simulating pure union and intersection blends. But they always deform primitives totally.
- To deform primitives locally after blending, blends with blending range parameters with C^1 continuity were developed in [8], [9]. The scale method [8] especially provides high-dimensional blends to blend over two primitives in a single blending operator. The range parameters limit the blend surface within specified regions, so the size of the transition of the resulting blend can be adjusted by varying range value and primitives are deformed locally. In addition, high-degree continuous blend with blending range parameters was also proposed in [6].

Regarding existing blends with blending range parameters, denoted as $B_k(f_1,...,f_k)$, in Constructive geometry, they make primitives behave like $Max(f_1,...,f_k)$ or $Min(f_1,...,f_k)$ in non-blending regions after blending. This causes the following problem:

When they are used as a new primitive in sequential blends, for example $B_2(B_k(f_1,...,f_k), f_{k+1})=1$ with range parameters r_1 and r_2 for B_k and f_{k+1}, $B_k(f_1,...,f_k)$ is unable to have an individual blending range control on the subsequent blends of f_1,..., and f_k with f_{k+1}. This is, primitives f_1,..., and f_k, always have similar sizes of subsequent blending surfaces with f_{k+1}, controlled only by r_1 for each of f_1,..., and f_k and r_2 for f_{k+1}.

To conquer this problem, this paper proposes new binary union blends, denoted as $B_{U2}(f_1, f_2)$, that not only provide range parameters r_1 and r_2 but also offer parameters m_1 and m_2 to make primitives behave like $Max(f_1^{1/m1}, f_2^{1/m2})$ or $Min(f_1^{1/m1}, f_2^{1/m2})$ in non-blending regions after blending. As a result, in sequential blends such as $B_2(B_{U2}(f_1,f_2), f_3)=1$ with range parameters r_1 and r_2 for B_{U2} and f_3, the sizes of subsequent blends of primitives f_1 and f_2 with f_3 can be adjusted by varying r_1, r_2 and m_1 for f_1 and r_1, r_2 and m_2 for f_2. Hence, m_1 and m_2 can be viewed as subsequent blend range parameters respectively for primitives f_1 and f_2, and the shape of blend surface $B_{U2}(f_1, f_2)$ will not change no matter what values m_1 and m_2 are set. Moreover, a generalized method is also proposed to develop the kind of blends stated above with C^1 continuity from an existing blend.

The remainder of this paper is organized as follows. Section II introduces related works and describes the problem more precisely. Section III presents the proposed blends. Section IV describes the applications of the proposed blends. Conclusion is given in Section V.

II. Related Works

In this section, implicit surfaces in Constructive geometry are defined first, then implicit blends are introduced, and finally their problem is described.

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A. Implicit Primitive Surface

In Constructive geometry [7], a primitive implicit surface is defined using a defining function $f(x,y,z):R^3 \rightarrow R_+$ and represented by a level surface

$$\{(x,y,z) \in R^3 \mid f(x,y,z)=1\}$$

which is denoted as $f_i(x,y,z)=1$ or $f_i=1$ for short in this paper and $R_+ \equiv [0, \infty]$. In the literature, many defining functions were proposed as listed below:

- Planes : $f(x,y,z)=|[x,y,z] \bullet \boldsymbol{v}|/a$,

where $\bullet$ means dot product, $\boldsymbol{v}$ is a unit normal vector toward the plane, $a>0$ controls the shortest Euclidean distance from the plane to the origin.

- Superellipsoids: $f(x,y,z)=(|x/a|^n+|y/b|^n+|z/c|^n)^{1/n}$

where parameters a, b and c decide the axial lengths of x, y, and z of the shape $f(x,y,z)=1$ [10].

- Superquadrics: $f(x,y,z)=((|x/a|^{n1}+|y/b|^{n1})^{n2/n1}+|z/c|^{n2})^{1/n2}$

where $n_1>1$ and $n_2>1$ are curvature parameters to control the shape of $f(x,y,z)=1$ [2], [5].

- Skeletal primitives: $f(x,y,z)=d/I_d$

where d is the shortest Euclidean distance from the point (x,y,z) to a given skeleton: a point, line segment, polygon or solid skeleton and I_d is a specified influential radius [3].

Fig. 1 displays some shapes of $f(x,y,z)=1$ defined using the above functions.

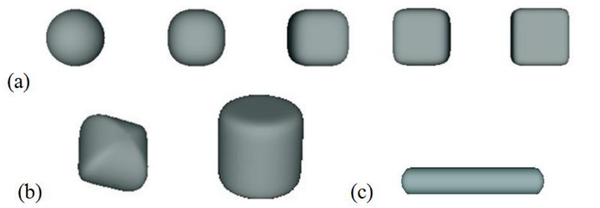

Figure 1. (a). Shapes of $(|x|^n+|y|^n+|z|^n)^{1/n}=1$ where n varies from 2, 3, 5, 10, and 20. (b). Shapes of two super-quadrics. (c). Shape of a skeletal primitive (circular cylinder).

B. Implicit Blends

Furthermore, a complex implicit surface is given by a blend $B_k(f_1,...,f_k)$, which is defined from primitive implicit surfaces $f_i(x,y,z)=1$, $i=1,...,k$, via a blending operator $B_k(x_1,..., x_k)$: $R_+^k \rightarrow R_+$ by:

$$B_k(f_1,...,f_k)=1$$

The boundary of $B_k(f_1,...,f_k)=1$ is called a blend surface. Blending operator $B_k(x_1,...,x_k)$ is to generate transitions tangent to primitives, which connects $f_i=1$ smoothly to erase sharp edges and corners as shown in Fig. 2(b).

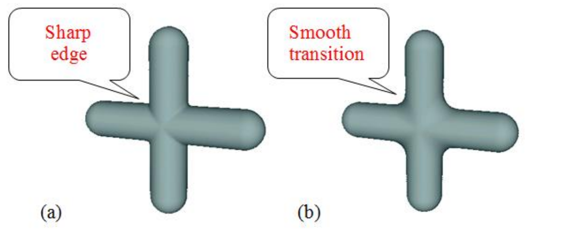

Figure 2. (a). A pure union Min of two cylinders, containing sharp edges. (b). A union blend of two cylinders, containing smooth transition generated automatically to remove sharp edges.

In the literature about Constructive geometry, boolean set blending operators, including, including union, intersection and difference, are listed below.

1) Pure union and intersection with C^0 continuity [7]

$B_{Ak}(x_1,...,x_k)=Min(x_1,...,x_k)$ and $B_{Sk}(x_1,...,x_k)=Max(x_1,...,x_k)$

They generate non-smooth surfaces as shown from the intersection in Fig. 3(b) because of only C^0 continuity.

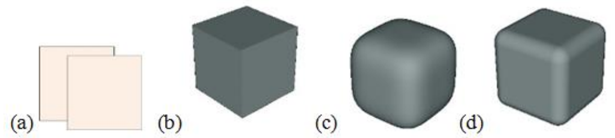

Figure 3. (a). Planes. (b)-(c). Intersection of 3 pairs of parallel planes in (a) by $B_{Sk}(f_1,f_2,f_3)=1$; (b). Sharp edges are generated because of using Max; (c). The planes are deformed totally because of using super-ellipsoidal intersection; (d). The planes are deformed locally because of using an intersection blend from the scale method.

2) Super-ellipsoidal union and intersection [7]

$B_{Ak}(x_1,...,x_k)=(x_1^{-p}+...+x_k^{-p})^{1/-p}$ and $B_{Sk}(x_1,...,x_k)=(x_1^{p}+...+x_k^{p})^{1/p}$

where p is a curvature parameter to adjust the shape of the transition of the resulting blend surface. These blends provide high-order continuity and hence generate smooth surfaces, but they always deform blended primitives totally as shown in Fig. 3(c).

Furthermore, a difference blending operator $B_{Dk}(x_1, x_2...,x_k)$ can be obtained from an intersection operator $B_{Sk}(x_1, x_2..., x_k)$ by

$$B_{Dk}(x_1, x_2...,x_k)=B_{Sk}(x_1, 1/x_2...,1/x_k),$$

$B_{Dk}(f_1, f_2...,f_k)=1$ subtracts $f_1=1$ from $f_2=1$,..., and $f_k=1$.

3) The scale method [8]

To make primitives deform locally like that in Fig. 3(d), the scale method was proposed to develop blends with range parameters. For example, given an existing union blending operator $H_k(x_1,\ldots,x_k)= \sum_{i=1}^{k}[1-x_i/r_i]_+^{p_i}$ -1=0 on $f_i \leq 0$ with range parameters r_i, $i=1,...,k$, the scale method can develop a union blending operators $B_{Ak}(x_1,\ldots,x_k)$ which maps R_+^k to R and possesses C^1 continuity for constructive geometry by :

$$B_{Ak}=\begin{cases} h_p & Min(x_1,\ldots,x_k)>0 \\ Min(x_1,\ldots,x_k) & otherwise \end{cases} \tag{1}$$

where $h_p \in T^{-1}(0)$,

$$T(h)=H_k(x_1/h-1,\ldots,x_k/h-1)=\sum_{i=1}^{k}[(1+r_i-x_i/h)/r_i]_+^{p_i}-1$$

and $r_i>0$ and $p_i>1$, for $i=1,...,k$.

Intersection blends from the scale method can be found in [8]. Equation (1) offers parameters p_i to adjust the shapes of the transitions; it also provide range parameters r_i to adjust the size of the transition and to make primitives deform locally after blending as shown on the region inside the box in Fig. 4. In addition, they can be used to generate sequential blends on overlapping blending regions, too.

In fact, $B_{Ak}(f_1,...,f_k)$ in (1) behaves similar to $Min(f_1,...,f_k)$ on non-blending regions, this leads to:

- A advantage:

Primitives f_1,..., and f_k do not change properties after blending or in later sequential blends.

- A disadvantage:

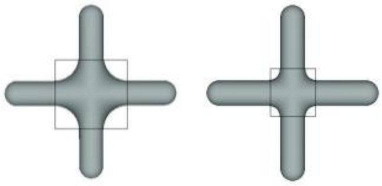

Figure 4. Union blends of two cylinders $B_{A2}(f_1,f_2)=1$ in (1), where the transitions have different sizes by setting ranges r_i different values to limit the transition located within the box and to make primitives f_1 and f_2 deform locally within the box, too.

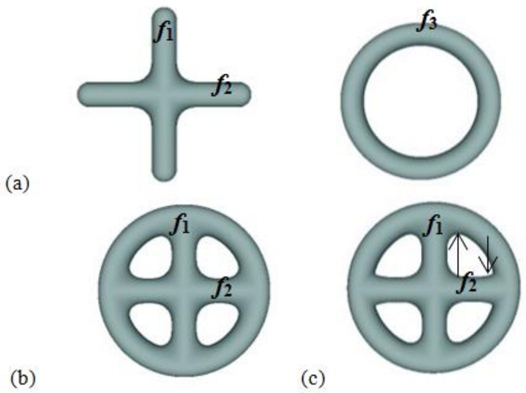

Figure 5. (a). Left: A union $B_{A2}(f_1, f_2)=1$ on cylinders; Right: A toroid $f_3=1$. (b). Sequential Union $B_{A2}(B_{A2}(f_1,f_2), f_3)=1$ by (1) from the scale method, where f_1 and f_2 always have similar subsequent blends with the toroid f_3. (c). Sequential unions $B_{A2}(B_{U2}(f_1,f_2), f_3)=1$ by the proposed blend from (2) in Section 3, which enables f_1 and f_2 to have different sizes of subsequent blends with the toroid f_3, pointed by arrows.

The advantage also incurs a disadvantage that in sequential blends such as $B_2(B_k(f_1, ..., f_k), f_{k+1})=1$ with blend range r_1 for B_k in B_2, primitives f_1, …, and f_k in B_k always have the same blending range r_1 to blend with f_{k+1} in B_2. Consequently, primitives f_1, ..., and f_k always have similar sizes of subsequent blend surfaces with f_{k+1}. This can be seen in Fig. 5(b) where f_1.and f_2 always have the same blend range with f_3 because of using (1) and so f_1.and f_2 have similar blends with f_3. That is, in $B_2(B_k(f_1, ..., f_k), f_{k+1})=1$, $B_{Ak}(f_1, ..., f_k)$ in (1) from the scale method does not offer f_1,..., and f_k subsequent blend range parameters to control their individual blend with f_{k+1} in B_2 without deforming the blending surfaces $B_k(f_1, …, f_k)=1$ like that in Fig. 5(c) where f_1 and f_2 are able to individually adjust the sizes of their subsequent blend surfaces with f_3 and have different sizes and the blend surface around the intersection region of cylinders keep unchanged.

To solve the problem, Section 3 proposes new blends that offer each primitive an individually adjustable range parameter to adjust the size of its subsequent blend with other primitives in later blends, and more importantly the blend surface never changes whatever values the parameters are set. For example, in $B_{A2}(B_{A2}(f_1, f_2), f_3)=1$ in Fig. 5(c), f_1 and f_2 in B_{A2} can have different sizes of blend surfaces with f_3 by setting the parameters different values and the blend surface $B_{A2}(f_1,f_2)=1$ keeps unchanged.

III. Blends with Individually Adjustable Primitives' Subsequent Blend Range Parameters

This section presents a generalized method that can develop a new binary union blend from an existing blend to conquer the problem stated at the end of Section II. From the method, a binary blend with C^1 continuity is developed later.

A. The Generalized Method

To solve a problem similar to that stated in Section II for Zero implicit surface $f_i(x,y,z)=0$, union blend $B_{T2}(f_1, f_2)=0$ in [11] enables primitives f_1 and f_2 to behave like f_1/m_1 and f_2/m_2 after blending and hence in sequential blends such as $B_k(B_{T2}(f_1, f_2), f_3,…, f_{k+1})=0$, one can individually adjust parameter m_i of B_{T2}, i=1 to 2, to control the sizes of f_i's successive blends with f_2, …, and f_k. However, $B_{T2}(f_1, f_2)$ unfortunately applies only to zero implicit surface $f_i(x,y,z)=0$, where $B_{T2}(f_1, f_2)$ and $f_i(x,y,z)$ all are defined to map to real-valued spaces, so it does not apply to constructive geometry $f_i(x,y,z)=1$ because $f_i(x,y,z)$ and their blending operators are required to map to non-negative spaces as stated in Section II. Even so, this paper still extends the idea in [11] and the scale method and then develop a generalized method which can create a new binary union blends, denoted as $B_{U2}(f_1, f_2)=1$ for Constructive geometry, from an existing blend such that $B_{U2}(f_1, f_2)$ enables primitives f_1 and f_2 to behave like $f_1^{1/m1}$ and $f_2^{1/m2}$ after blending. As a result, each m_i of B_{U2}, i=1 to 2, can be viewed as a f_i's subsequent blending range parameter to control the size of f_i's subsequent blend. The generalized method includes two steps, described as follows:

Step (1): Choose a base curve $H_2(x_1,x_2)=0$ defined piecewise by an existing union operator $H_{A2}(x_1,x_2)=0$ on f_i=0, with blending range parameters r_1 and r_2 tangent to $Min(x_1,x_2)-1=0$. More precisely, the former curve is an arc-shaped curve tangent to the latter one at points (1, $1+r_2$) and ($1+r_1$, 1), as in Fig. 6.

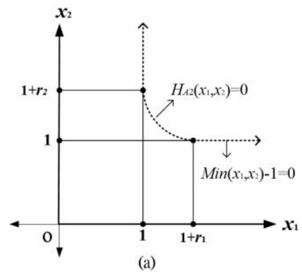

Figure 6. Base curve $H_2(x_1, x_2)=0$.

Step (2): Define a union operator $B_{U2}(x_1,x_2):R^2\rightarrow R_+$ with blending range parameters $r_1>0$ and $r_2>0$ and parameters $m_1>0$ and $m_2>0$ from the base curve $H_2(x_1,x_2)=0$ in **Step (1)** by:

$$B_{U2}(x_1,x_2)=\begin{cases} {x_1}^{1/m1} & x_2\geq h_1 \\ {x_2}^{1/m2} & x_1\geq h_2 \\ h_p & \text{Otherwise}\end{cases} \tag{2}$$

where h_p is the root h of the equation $T(h)=0$ for any (x_1, x_2), and

$$T(h)=H_{A2}(x_1/(h^{m_1}),\ x_2/(h^{m_2})),$$
$$h_1=(1+r_2)(\,x_1^{m2/m1})\ \text{and}\ h_2=(1+r_1)(\,x_2^{m1/m2}).$$

Some properties of $B_{U2}(f_1(x,y,z),\ f_2(x,y,z))$ in (2) are analyzed in the following:

(1). Whatever positive values m_1 and m_2 are set, the blending curve $B_{U2}(x_1,x_2)=1$, i.e. $h_p=1$, is always the same as the base curve $H_2(x_1,x_2)=0$ with blending ranges r_1 and r_2 and hence the blend surface $B_{U2}(f_1,f_2)=1$ is always the same union blend of surfaces $f_1=1$ and $f_2=1$, whose shape remains unchanged. This is shown from the union blend of intersecting cylinders $B_{U2}(f_1,f_2)=1$ in Fig. 7, whose shape remains unchanged for m=0.1, 0.5 0.8, 1, 1.4 to 1.8.

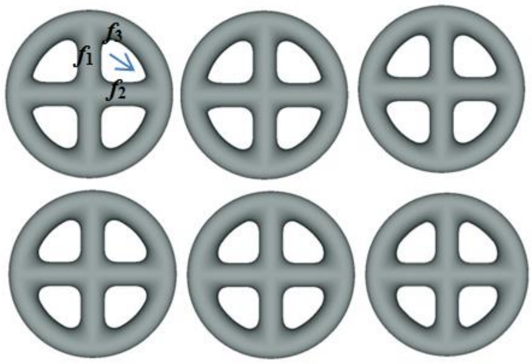

Figure 7. Sequential unions $B_{A2}(B_{U2}(f_1,f_2),\ f_3)=1$ by (2) of $B_{U2}(f_1,\ f_2)=1$, intersecting cylinders, and $f_3=1$, a toroid. Varying m_2 in $B_{U2}(f_1,\ f_2)$ for f_2 from 0.1, 0.5 0.8, 1, 1.4 to 1.8 make the subsequent blends of f_2 with f_3 enlarging gradually without changing the shape of the transition on the intersecting region between cylinders as shown in surfaces from top left to bottom right.

(2). Every level curve of $B_{U2}(x_1,x_2)=h,\ h>0$, can be viewed as the base curve $H_2(x_1,x_2)=0$ whose x_1 axis and x_2 axis are scaled by h^{m1} and h^{m2}, respectively. This tells that every surface $B_{U2}(f_1,\ f_2)=h,\ h>0$, can be considered as a union blend surface of surfaces $f_1/h^{m_1}=1$ and $f_2/h^{m_2}=1$ with blending ranges r_i, i=1 and 2. Consequently, $B_{U2}(f_1,f_2)$ can be used as a new primitive in other blends to generate sequential blends with overlapping blending regions. Especially, in non-blending regions $B_{U2}(f_1,f_2)$ is the same as

$$Min(f_1^{(1/m_1)}, f_2^{(1/m_2)})$$

That is, $f_1$and f_2 in $B_{U2}(f_1,\ f_2)=l$ in non-blending regions behaves the same as

$$f_i^{(1/m_i)}=l \equiv\ f_i=l^{m_i}$$

It follows that if $m_i<m_j$ and $l>1$, level surface $f^{(1/m_i)}=l$ dilates less than $f^{(1/m_j)}=l$ as shown from f_2 in Fig. 8; and if $m_i<m_j$ and $l<1$, $f^{(1/m_i)}=l$ shrinks less as shown from f_2 in Fig. 9.

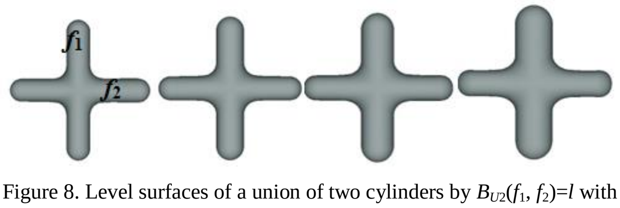

Figure 8. Level surfaces of a union of two cylinders by $B_{U2}(f_1,\ f_2)=l$ with $m_1=1$ and $m_2=0.25$ for l=1, 1.2, 1.4 and 1.6; On all of them, f_2 dilates less than f_1 because of $m_2=0.25$.

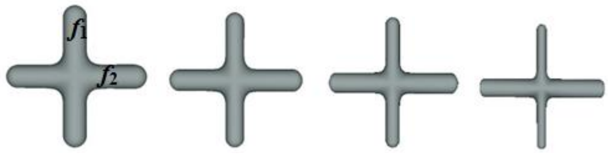

Figure 9. Level surfaces of a union of two cylinders by $B_{U2}(f_1,\ f_2)=l$ with $m_1=1$ and $m_2=0.25$ for l=1, 0.8, 0.6 and 0.4; on all of them, f_2 shrinks less than f_1 because of $m_2=0.25$.

(3). When B_{U2} is used as a new primitive in another added-material blend, such as $B_{A2}(B_{U2}(f_1,f_2),\ f_3)=1$ where the blend range for B_{U2} in B_{A2} is set r_a, varying m_1 and m_2 in B_{U2} enables primitives f_1 and f_2 to have different blending ranges

$$(1+r_a)^{m_i}\text{-}1,\ i\text{=1 and 2}$$

to blend with f_3 in B_{U2}.

As m_i increases from 1, the blending range of f_i with f_3 is getting larger from r_a and so the resulting (added-material) blend surface is getting bigger, too. As m_i decrease from 1, the blending range of f_i with f_3 is getting smaller from r_a to 0 so the resulting blend surface gets smaller. These can be found from the blend pointed by an arrow which is getting larger as m_i increases in Fig. 7.

(5). When B_{U2} is used as a new primitive in another subtracted-material blend, such as $B_{D2}(B_{U2}(f_1,f_2),\ f_3)=1$ where the blend ranges for B_{U2} in B_{D2} are set r_a, varying m_1 and m_2 in B_{U2} enables primitives f_1 and f_2 to have different blending ranges

$$1\text{-}(1\text{-}r_a)^{m_i},\ i\text{=1 and 2}$$

to blend with f_3 in B_{D2}.

As m_i increases from 1, the blending range of f_i with f_3 increases from r_a and so the resulting (subtracted-material) blend surface shrinks more. As m_i decrease from 1, the blending range of f_i with f_3 decreases from r_a and so the resulting blend shrinks less.

In addition, when m_i i=1 and 2, both are set 1, Equation (2) is similar to binary blends of the scale method. However, the major deference between $B_{U2}(f_1,f_2)$ in (2) and $B_{A2}(f_1,f_2)$ in (1) is that when B_{U2} is used as a new primitive in sequential blends, m_1 and m_2 in B_{U2} can be viewed as additional range parameters to adjust the subsequent added-material or subtracted-material blends of primitive f_1 and f_2 with other primitives, which solves the problem stated in the end of Section II.

B. Differentiable Blending Operators

To ensure that $B_{U2}(x_1,x_2)$ in (2) is exactly a function, we need to find an arc-shaped curve $H_{A2}(x_1,x_2)$ such that there exists a unique root h_p of $T(h)=0$ for any (x_1,x_2). In fact, many existing union blending operators on zero implicit surfaces satisfy this requirement and so can be used as a base curve in **Step (1)** in Section III.A, such as Hoffmann's conic blend [12], Middleditch's elliptic blends [13] and super-ellipsoidal blends [14]. Now, Hoffmann's conic blend $H_H(x_1,x_2)=0$ is applied to develop $B_{U2}(x_1,x_2)$ and it is represented by

$$H_H(x_1,x_2)=r_2^2x_1^2+r_1^2x_2^2+r_2^2r_1^2\text{-}2r_2^2r_1x_1\text{-}2r_1^2r_2x_2+2px_1x_2$$

where $-\infty<p<r_1r_2$. Thus, $H_{A2}(x_1,x_2)=0$ in (2) can be given by $H_H(x_1-1, x_2-1)=0$. Afterwards, a binary conic blending operator B_{U2} with range parameters r_1 and r_2 and curvature parameter p and parameters m_1 and m_2 to adjust the subsequent blends of f_1 and f_2 is given by:

$$B_{A2}(x_1,x_2) = \begin{cases} {x_1}^{1/m1} & x_2 \geq h_2 \\ {x_2}^{1/m2} & x_1 \geq h_2 \\ h_p & otherwise \end{cases} \quad (3)$$

where

$$h_1=(1+r_2)({x_1}^{m2/m1}),\ h_2=(1+r_1)({x_2}^{m1/m2})$$

and h_p is the root h of the equation $T(h)=0$ for any (x_1, x_2):

$$T(h)=H_H(x_1/(h^{m1})-1,\ x_2/(h^{m2})-1)$$

The root h_p of the equation $T(h)=0$ can be solved by Newton-Ralphson numerical method and the value $h=Min({x_1}^{1/m_1}, {x_2}^{1/m_2})$ is the initial guess for solving h_p.

C. Bulge Elimination

Similar to that in [8], [14], [15], union blend $B_{U2}(f_1, f_2)=1$ from (2) or (3) is able to perform bulge elimination by replacing ranges r_1 and r_2 with positive position functions R_i, $i=1$, 2 to make the values of r_1 and r_2 approach 0 around the region $\cos\theta\cong 1$ by

$$R_i(x,y,z)=r_i(G(x,y,z)+\omega)$$

where $G(x,y,z)$ maps R^3 to [0, 2] or R^3 to [0, 1], depending on which one of the following is used:

$$G(x,y,z)=1-cos\theta \quad (4)$$

$$G(x,y,z)=\begin{cases}(1-cos\theta^2)^n & cos\theta \geq 0 \\ 1 & \text{Otherwise}\end{cases} \quad (5)$$

And θ is the angle between the gradients of f_1 and f_2 at the point (x,y,z), $n>1$ and $\omega \cong 0$. $G(x,y,z)$ in (4) maps to [0, 2]; $G(x,y,z)$ in (5) maps to [0, 1] which can be used to avoid unwanted blends or to avoid changing the topology of a union blend of closed primitives like those stated in [15], [16].

However, on bulge elimination, $B_{U2}(f_1,f_2)$ along with R_i is almost similar to a pure union $Min({f_1}^{(1/m_1)}, {f_2}^{(1/m_2)})$ around the region $\cos\theta\cong 1$, where the value of G is close to zero. This means when $m_2\neq m_1$, $\|\nabla({f_1}^{(1/m_1)})\|$ might vary dramatically to $\|\nabla({f_2}^{(1/m_2)})\|$ around $\cos\theta\cong 1$, and hence might cause computational errors in polygon-generation process. To reduce dramatic change in gradients, both the values of m_1 and m_2 are required to approach 1 around $\cos\theta\cong 1$, by replacing parameters m_i, $i=1$ and 2, with position functions $M_i(x,y,z)$, which maps ${R_+}^3$ to $(1, m_i)$ for $m_i>1$ or $(m_i, 1)$ for $m_i>1$, by

$$M_i=1+(m_i-1)V(x,y,z)$$

where $V(x,y,z)$ maps to [0,1], given by

$$V(x,y,z)=\begin{cases}(1-cos\theta^2)^n & cos\theta \geq 0 \\ 1 & \text{Otherwise}\end{cases}$$

and $cos\theta$ is the same as in $R_i(x,y,z)$ above and parameter n controls the rate M_i varies from 1 to m_i at.

IV. Applications

$B_{U2}(f_1, f_2)$ in (3) can be integrated with other existing blends such as those of the scale method and super-ellipsoidal blends. Its major applications are described below.

A. Application to an Added-Material Blend

When $B_{U2}(f_1, f_2)$ is used as new primitives in an added-material blend, for example in $B_{A2}(B_{U2}(f_1,f_2),f_3)$, changing m_1 and m_2 of B_{U2} can adjust the individual blend range of f_1 and f_2 with f_3 as shown from the blend of f_2 with f_3 in Fig. 9. Fig. 10(a) displays two sets of four connecting cylinders defined by $f_a=B_{A2}(f_4,B_{A2}(f_3,B_{A2}(f_2, f_1))$ and $f_b=B_{A2}(f_8,B_{A2}(f_7,B_{A2}(f_6, f_5))$, respectively. The blend surfaces of union $B_{A2}(f_a, f_b)=1$ on the intersecting regions in Fig. 10(b) always have similar sizes because of defining f_a and f_b using B_{A2} in (1). However, the three surfaces from left to right in Fig. 10(c) indicates that if $f_a=B_{U2}(f_4, B_{U2}(f_3, B_{U2}(f_2,f_1))$ and $f_b=B_{U2}(f_8,B_{U2}(f_7,B_{U2}(f_6,f_5))$ defined by (3) are used instead, then union $B_{A2}(f_a, f_b)$ can individually adjust the sizes of both the blend surfaces on the intersecting regions by setting both m_1 for f_4 in f_a and f_8 in f_b 0.2, 1.2 and 1.8, respectively.

B. Application to a Subtracted-Material Blend

When $B_{U2}(f_1, f_2)$ is used as a subtracting or a subtracted operand in a difference blend, for example $B_{D2}(B_{U2}(f_1, f_2), B_{U2}(f_3, f_4))$, changing m_1 and m_2 of $B_{U2}(f_1, f_2)$ can adjust the individual blend range (or the size) of the subtracted-material blends of f_1 and f_2 from $B_{U2}(f_3, f_4)$. This can be seen in Fig. 11, which displays a difference blend of bigger intersecting cylinders $B_{U2}(f_1, f_2)$ in Fig. 11(a) from another one $B_{U2}(f_3, f_4)$ in Fig. 11(b), defined by $B_{D2}(B_{U2}(f_1, f_2), B_{U2}(f_3, f_4))$. Setting 1.3 and 0.25 to m_1 and m_2 of both B_{U2} makes the edges of the walls between f_2 and f_4 thinner than that of the wall between f_1 and f_3, as shown in Fig. 11(d). Fig. 11(c) shows the same blend as Fig. 11(d) does except B_{A2} is used instead of B_{U2} and so all the edges on it always have similar sizes.

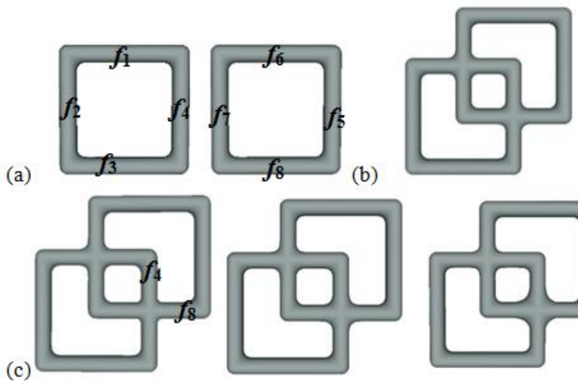

Figure 10. (a). Two sets of four connecting cylinders defined by $f_a=B_{A2}(f_4,B_{A2}(f_3,B_{A2}(f_2,f_1))$ and $f_b=B_{A2}(f_8,B_{A2}(f_7,B_{A2}(f_6,f_5))$. (b). Union $B_{A2}(f_a, f_b)=1$ where the blend surfaces on both of the intersecting regions always have similar sizes because of defining f_a and f_b using B_{A2}. (c). Unions $B_{A2}(f_a, f_b)=1$, $f_a=B_{U2}(f_4, B_{U2}(f_3, B_{U2}(f_2,f_1))$ and $f_b=B_{U2}(f_8, B_{U2}(f_7, B_{U2}(f_6,f_5))$; the sizes of the intersecting cylinders of f_4 and f_8 on the surfaces from left to right are different because of defining f_a and f_b using B_{U2} and setting m_1 for both f_4 and f_8 0.2, 1,2 and 1.8, respectively.

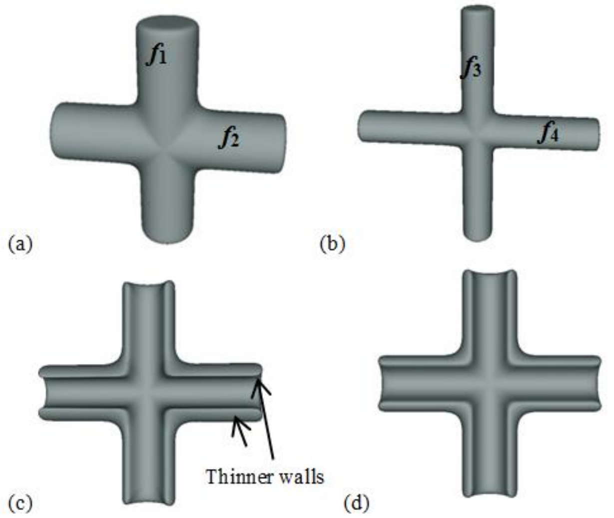

Figure 11. (a). Bigger intersecting cylinders by $B_{U2}(f_1, f_2)$. (b). Smaller intersecting cylinders by $B_{U2}(f_3, f_4)$. (c). Difference of the blend in (a) from the one in (b) by $B_{D2}(B_{U2}(f_1, f_2), B_{U2}(f_3, f_4))$ where m_1=1.3 and m_2=0.25 for both B_{U2} and hence the edges of the walls between f_2 and f_4, pointed by arrows, are thinner than the others. (d). The same difference as in (c) except that m_1=1 and m_2=1 and hence all the edges have similar sizes.

V. Conclusion

In Ricci's constructive geometry, existing blends with range parameters behave like pure union or intersection, *Max* or *Min*, in non-blending regions. This implies that their primitives always have similar sizes of subsequent blending surfaces in sequential blends. In order to solve the problem, this paper has proposed a generalized method that is able to transform an existing union blend into a new binary union blend that

- Provide primitives additional parameters m_1 and m_2 to individually adjust the blending range and the size of each primitive's subsequent blend in sequential blends, and especially the blend surface does not change no matter what values the parameters are set.
- Provide primitives range parameters to adjust the size of the transition of each primitive's blend surface and to make the transition generated in a specified region such that primitives deform locally.
- Are C^1 continuous everywhere and so can be used as a new primitive in other blends to generate sequential blends with overlapping blending regions.

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