Piecewise Union Blend with Adjustable Primitive’s Subsequent Blend Range Parameters in Constructive Geometry

Pi-Chung Hsu
Department of Information Management, Shu-Te University, Kaohsiung City, Taiwan
Email: pichung@stu.edu.tw

Abstract—In constructive geometry, implicit blends with blend range parameters are important in constructing a complex surface because they limit the blend surface within specified regions and deform primitive locally after blending. However, because they behave like Max or Min blend in non-blending regions, their primitives always have similar sizes of subsequent blend surfaces when used as a primitive in sequential blends. To solve this problem, this paper proposes union blends that provide each primitive an additional parameter to adjust a primitive’s subsequent blending range in later blends; thus, primitives are able to individually adjust the sizes of their subsequent blends by varying the values of the additional parameters. In addition, a generalized method is proposed to develop the proposed union blends with $C^1$ continuity from an existing blend.

IndexTerms—implicit surface, implicit blends, constructive geometry

I. INTRODUCTION

In Constructive geometry of implicit surfaces, primitive implicit surfaces are defined as 1 level surface of a defining function. Defining functions determine the shapes of primitives, which can be found in [1]-[6]. A complex surface is then constructed easily from primitive surfaces, such as planes, ellipsoids, skeletal primitives, etc. by implicit blends. A blend connects and joins primitives with transitions generated automatically to smooth out unwanted sharp edges and creases. Existing blends in Constructive geometry in the literature are reviewed as follows:

- Max and Min functions offer pure union and intersection [7]. Because they are $C^0$ continuous only, they always generate non-smooth surfaces.
- To deform primitives locally after blending, blends with blending range parameters with $C^1$ continuity were developed in [8], [9]. The scale method [8] especially provides high-dimensional blends to blend over two primitives in a single blending operator. The range parameters limit the blend surface within specified regions, so the size of the transition of the resulting blend can be adjusted by varying range value and primitives are deformed locally. In addition, high-degree continuous blend with blending range parameters was also proposed in [6].

Regarding existing blends with blending range parameters, denoted as $B_i(f_1,...,f_k)$, in Constructive geometry, they make primitives behave like Max$(f_1,...,f_k)$ or Min$(f_1,...,f_k)$ in non-blending regions after blending. This causes the following problem:

When they are used as a new primitive in sequential blends, for example $B_2(B_i(f_1,...,f_k), f_{k+1})=1$ with range parameters $r_1$ and $r_2$ for $B_i$ and $f_{k+1}$, $B_i(f_1,...,f_k)$ is unable to have an individual blending range control on the subsequent blends of $f_1,...,$ and $f_k$ with $f_{k+1}$. This is, primitives $f_1,...,$ and $f_k$ always have similar sizes of subsequent blending surfaces with $f_{k+1}$, controlled only by $r_1$ for each of $f_1,...,$ and $f_2$ and $r_2$ for $f_{k+1}$.

To conquer this problem, this paper proposes new binary union blends, denoted as $B_{uv}(f_1, f_2)$, that not only provide range parameters $r_1$ and $r_2$ but also offer parameters $m_1$ and $m_2$ to make primitives behave like Max$(f_1^{1m_1}, f_2^{1m_2})$ or Min$(f_1^{1m_1}, f_2^{1m_2})$ in non-blending regions after blending. As a result, in sequential blends such as $B_2(B_{uv}(f_1,f_2), f_3)=1$ with range parameters $r_1$ and $r_2$ for $B_{uv}$ and $f_3$, the sizes of subsequent blends of primitives $f_1$ and $f_2$ with $f_3$ can be adjusted by varying $r_1$, $r_2$ and $m_1$ for $f_1$ and $r_1$, $r_2$ and $m_2$ for $f_2$. Hence, $m_1$ and $m_2$ can be viewed as subsequent blend range parameters respectively for primitives $f_1$ and $f_2$ and the shape of blend surface $B_{uv}(f_1, f_2)$ will not change no matter what values $m_1$ and $m_2$ are set. Moreover, a generalized method is also proposed to develop the kind of blends stated above with $C^1$ continuity from an existing blend.

The remainder of this paper is organized as follows. Section II introduces related works and describes the problem more precisely. Section III presents the proposed blends. Section IV describes the applications of the proposed blends. Conclusion is given in Section V.

II. RELATED WORKS

In this section, implicit surfaces in Constructive geometry are defined first, then implicit blends are introduced, and finally their problem is described.
A. Implicit Primitive Surface

In Constructive geometry [7], a primitive implicit surface is defined using a defining function \( f(x,y,z): \mathbb{R}^3 \rightarrow \mathbb{R} \), and represented by a level surface

\[
\{ (x,y,z) \in \mathbb{R}^3 | f(x,y,z)=1 \}
\]

which is denoted as \( f(x,y,z) = 1 \) or \( f = 1 \) for short in this paper and \( R_1 = \{0, \infty \} \). In the literature, many defining functions were proposed as listed below:

- **Planes**: \( f(x,y,z) = ||x,y,z|| \mathbf{w}f \)
  where \( \mathbf{w} \) means dot product, \( \mathbf{w} \) is a unit normal vector toward the plane, \( a \neq 0 \) controls the shortest Euclidean distance from the plane to the origin.

- **Superellipsoids**: \( f(x,y,z)=\left(|x/a|^n+|y/b|^n+|z/c|^n\right)^{1/n} \)
  where parameters \( a \) and \( b \) determine the axial lengths of \( x \), \( y \), and \( z \) of the shape \( f(x,y,z)=1 \) [10].

- **Superquadrics**: \( f(x,y,z)=\left(|x|^n+|y|^n+|z|^n\right)^{1/n} \)
  which are defined using a defining function \( f = 1 \) smoothly to the transition of the resulting blend surface. These blends provide high-order continuity and hence generate smooth surfaces, but they always deform blended primitives totally as shown in Fig. 3(c).

- **Skeletal primitives**: \( f(x,y,z)=d/I_d \)
  where \( d \) is the shortest Euclidean distance from the point \( (x,y,z) \) to a given skeleton: a point, line segment, polygon or solid skeleton and \( I_d \) is a specified influential radius [3].

Fig. 1 displays some shapes of \( f(x,y,z)=1 \) defined using the above functions.

B. Implicit Blends

Furthermore, a complex implicit surface is given by a blend \( B_k (f_1, \ldots, f_k) \), which is defined from primitive implicit surfaces \( f(x,y,z)=1 \), \( i=1, \ldots, k \), via a blending operator \( B_k(x_1, \ldots, x_k) : R_k^k \rightarrow R_1 \) by:

\[ B_k(f_1, \ldots, f_k)=1 \]

The boundary of \( B_k(f_1, \ldots, f_k)=1 \) is called a blend surface. Blending operator \( B_k(x_1, \ldots, x_k) \) is to generate transitions tangent to primitives, which connects \( f_i=1 \) smoothly to erase sharp edges and corners as shown in Fig. 2(b).

In the literature about Constructive geometry, boolean set blending operators, including, including union, intersection and difference, are listed below.

1) **Pure union and intersection with \( C^2 \) continuity** [7]

\[
B_A(x_1, \ldots, x_k)=\min(x_1, \ldots, x_k) \quad \text{and} \quad B_S(x_1, \ldots, x_k)=\max(x_1, \ldots, x_k)
\]

They generate non-smooth surfaces as shown from the intersection in Fig. 3(b) because of only \( C^0 \) continuity.

![Figure 3](image)

Figure 3. (a). Planes. (b)-(c). Intersection of 3 pairs of parallel planes in (a) by \( B_A(f_1, f_2, f_3)=1 \); (b). Sharp edges are generated because of using Max; (c). The planes are deformed totally because of using super-ellipsoidal intersection; (d). The planes are deformed locally because of using an intersection blend from the scale method.

2) **Super-ellipsoidal union and intersection** [7]

\[
B_k(x_1, \ldots, x_k)=\left(x_1^p + \ldots + x_k^p\right)^{1/p} \quad \text{and} \quad B_k(x_1, \ldots, x_k)=\left(x_1^p + \ldots + x_k^p\right)^{1/p}
\]

where \( p \) is a curvature parameter to adjust the shape of the transition of the resulting blend surface. These blends provide high-order continuity and hence generate smooth surfaces, but they always deform blended primitives totally as shown in Fig. 3(c).

Furthermore, a difference blending operator \( B_D(x_1, \ldots, x_k) \) can be obtained from an intersection operator \( B_S(x_1, \ldots, x_k) \) by

\[
B_D(x_1, x_2, \ldots, x_k)=B_S(x_1, 1/x_2, \ldots, 1/x_k).
\]

3) **The scale method** [8]

To make primitives deform locally like that in Fig. 3(d), the scale method was proposed to develop blends with range parameters. For example, given an existing union blending operator \( H_k(x_1, \ldots, x_k)=\sum_{i=1}^k \left[1 - x_i / r_i\right]^{p_i} -1 \) on \( f \leq 0 \) with range parameters \( r_i, i=1, \ldots, k \), the scale method can develop a union blending operators \( B_k(x_1, \ldots, x_k) \) which maps \( R_k \) to \( R \) and possesses \( C^1 \) continuity for constructive geometry by:

\[
B_{Ak} = \begin{cases} 
\begin{array}{ll}
\frac{h_p}{M(x_1, \ldots, x_k)} & \text{Min}(x_1, \ldots, x_k) > 0 \\
\frac{1}{M(x_1, \ldots, x_k)} & \text{otherwise}
\end{array}
\end{cases}
\]

where \( h_p \in T^{-1}(0) \),

\[
T(h)=H_k(x_1/h, \ldots, x_k/h)-1 = \sum_{i=1}^k \left((1 + r_i - x_i/h)/r_i\right)^{p_i} -1
\]

and \( r_i > 0 \) and \( p_i > 1 \), for \( i=1, \ldots, k \).

Intersection blends from the scale method can be found in [8]. Equation (1) offers parameters \( p_i \) to adjust the shapes of the transitions; it also provide range parameters \( r_i \) to adjust the size of the transition and to make primitives locally after blending as shown on the region inside the box in Fig. 4. In addition, they can be used to generate sequential blends on overlapping blending regions, too.

In fact, \( B_{Ak}(f_1, \ldots, f_k) \) in (1) behaves similar to \( \text{Min}(f_1, \ldots, f_k) \) on non-blending regions, this leads to:
• A advantage:
Primitives $f_1$, ..., and $f_k$ do not change properties after blending or in later sequential blends.

• A disadvantage:

![Figure 4. Union blends of two cylinders $B_{c2}(f_1, f_2)=1$ in (1), where the transitions have different sizes by setting ranges $r_i$ different values to limit the transition located within the box and to make primitives $f_1$ and $f_2$ deform locally within the box, too.](image)

![Figure 5. (a). Left: A union $B_{c2}(f_1, f_2)=1$ on cylinders; Right: A toroid $f_2=1$. (b). Sequential Union $B_{c2}(B_{c2}(f_1, f_2), f_3)=1$ by (1) from the scale method, where $f_1$ and $f_2$ always have similar subsequent blends with the toroid $f_3$. (c). Sequential unions $B_{c2}(B_{c2}(f_1, f_2), f_3)=1$ by the proposed blend from (2) in Section 3, which enables $f_1$ and $f_2$ to have different sizes of subsequent blends with the toroid $f_3$, pointed by arrows.](image)

The advantage also incurs a disadvantage that in sequential blends such as $B_2(B_2(f_1, ..., f_k), f_{k+1})=1$ with blend range $r_1$ for $B_1$ in $B_2$, primitives $f_1$, ..., and $f_k$ in $B_k$ always have the same blend range $r_1$ to blend with $f_{k+1}$ in $B_2$. Consequently, primitives $f_1$, ..., and $f_k$ always have similar sizes of subsequent blend surfaces with $f_{k+1}$. This can be seen in Fig. 5(b) where $f_1$ and $f_2$ always have the same blend range with $f_3$ because of using (1) and so $f_1$ and $f_2$ have similar blends with $f_3$. That is, in $B_2(B_2(f_1, ..., f_k), f_3)=1$, $B_{c2}(f_1, ..., f_k)$ in (1) from the scale method does not offer $f_{k+1}$ and $f_k$ subsequent blend range parameters to control their individual blend with $f_{k+1}$ in $B_2$ without deforming the blending surfaces $B_2(f_1, ..., f_k)=1$ like that in Fig. 5(c) where $f_1$ and $f_2$ are able to individually adjust the sizes of their subsequent blend surfaces with $f_3$ and have different sizes and the blend surface around the intersection region of cylinders keeps unchanged.

To solve the problem, Section 3 proposes new blends that offer each primitive an individually adjustable range parameter to adjust the size of its subsequent blend with other primitives in later blends, and more importantly the blend surface never changes whatever values the parameters are set. For example, in $B_{c2}(B_{c2}(f_1, f_2), f_3)=1$ in Fig. 5(c), $f_1$ and $f_2$ in $B_{c2}$ can have different sizes of blend surfaces with $f_3$ by setting the parameters different values and the blend surface $B_{c2}(f_1, f_2)=1$ keeps unchanged.

### III. Blends with Individually Adjustable Primitives’ Subsequent Blend Range Parameters

This section presents a generalized method that can develop a new binary union blend from an existing blend to conquer the problem stated at the end of Section II. From the method, a binary blend with $C^0$ continuity is developed later.

#### A. The Generalized Method

To solve a problem similar to that stated in Section II for Zero implicit surface $f_i(x,y,z)=0$, union blend $B_{c2}(f_1, f_2)=0$ in [11] enables primitives $f_1$ and $f_2$ to behave like $f_1/m_1$ and $f_2/m_2$ after blending and hence in sequential blends such as $B_3(B_2(f_1, f_2), f_3, ..., f_{k+1})=0$, one can individually adjust parameter $m_i$ of $B_2$, $i=1$ to 2, to control the sizes of $f_i$’s successive blends with $f_2$, ..., and $f_k$. However, $B_2(f_1, f_2)$ unfortunately applies only to zero implicit surface $f_i(x,y,z)=0$, where $B_2(f_1, f_2)$ and $f_i(x,y,z)$ all are defined to map to real-valued spaces, so it does not apply to construct geometry $f_i(x,y,z)=1$ because $f_i(x,y,z)$ and their blending operators are required to map to non-negative spaces as stated in Section II. Even so, this paper still extends the idea in [11] and the scale method and then develop a generalized method which can create a new binary union blends, denoted as $B_{c2}(f_1, f_2)=1$ for Constructive geometry, from an existing blend such that $B_{c2}(f_1, f_2)$ enables primitives $f_1$ and $f_2$ to behave like $f_1^{lim_1}$ and $f_2^{lim_2}$ after blending. As a result, each $m_i$ of $B_{c2}$, $i=1$ to 2, can be viewed as a $f_i$’s subsequent blending range parameter to control the size of $f_i$’s subsequent blend. The generalized method includes two steps, described as follows:

**Step (1):** Choose a base curve $H(x, x_0)=0$ defined piecewise by an existing union operator $H_{c2}(x, x_0)=0$ on $f_0=0$, with blending range parameters $r_1$ and $r_2$ tangent to $Min(x, x_0)=1$. More precisely, the former curve is an arc-shaped curve tangent to the latter one at points $(1, 1+r_2)$ and $(1+r_1, 1)$, as in Fig. 6.

![Figure 6. Base curve $H_0(x, x_0)=0$.](image)

**Step (2):** Define a union operator $B_{c2}(x_1, x_2): R^2 \to R_+$ with blending range parameters $r_1>0$ and $r_2>0$ and parameters $m_1>0$ and $m_2>0$ from the base curve $H_0(x, x_0)=0$ in **Step (1)** by:

$$B_{c2}(x_1, x_2) = \begin{cases} x_1^{1/m_1} & x_2 \geq h_1 \\ x_2^{1/m_2} & x_1 \geq h_2 \\ h_0 & \text{Otherwise} \end{cases}$$

where $h_0$ is the root $h$ of the equation $T(h)=0$ for any $(x_1, x_2)$, and
Some properties of $B_U(f_1(x,y,z), f_2(x,y,z))$ in (2) are analyzed in the following.

(1). Whatever positive values $m_1$ and $m_2$ are set, the blending curve $B_U(x_1,x_2)=1$, i.e., $h_0=1$, is always the same as the base curve $H_2(x_1,x_2)=0$ with blending ranges $r_1$ and $r_2$ and hence the blend surface $B_U(f_1,f_2)=1$ is always the same union blend of surfaces $f_1=1$ and $f_2=1$, whose shape remains unchanged. This is shown from the union blend of intersecting cylinders $B_U(f_1,f_2)=1$ in Fig. 7, whose shape remains unchanged for $m=0.1, 0.5, 0.8, 1, 1.4$ to 1.8.

(2). Every level curve of $B_U(x_1,x_2)=h$, $h>0$, can be viewed as the base curve $H(x_1,x_2)=0$ whose $x_1$ axis and $x_2$ axis are scaled by $h^{m_1}$ and $h^{m_2}$, respectively. This tells that every surface $B_U(f_1,f_2)=h$, $h>0$, can be considered as a union blend surface of surfaces $f_1 h^{m_1}=1$ and $f_2 h^{m_2}=1$ with blending ranges $r$, $i=1$ and 2. Consequently, $B_U(f_1,f_2)$ can be used as a new primitive in other blends to generate sequential blends with overlapping blending regions. Especially, in non-blending regions $B_U(f_1,f_2)$ is the same as

$$\text{Min}(f_1^{(1/m_1)}, f_2^{(1/m_2)})$$

That is, $f_1$ and $f_2$ in $B_U(f_1,f_2)=1$ in non-blending regions behaves the same as

$$f_1^{(1/m_1)}=f_2^{(1/m_2)}$$

It follows that if $m_1<m_2$ and $l>1$, level surface $f_1^{(1/m_1)}=1$ dilates less than $f_2^{(1/m_2)}=1$ as shown from $f_2$ in Fig. 8; and if $m_1>m_2$ and $l<1$, $f_1^{(1/m_1)}=1$ shrinks less as shown from $f_2$ in Fig. 9.

(3). When $B_U$ is used as a new primitive in another added-material blend, such as $B_A(B_U(f_1,f_2), f_3)=1$ where the blend range for $B_U$ in $B_A$ is set $r_m$, varying $m_1$ and $m_2$ in $B_U$ enables primitives $f_1$ and $f_2$ to have different blending ranges

$$(1+r_m)^{m_i}=1, i=1 \text{ and } 2$$

As $m_i$ increases from 1, the blending range of $f_i$ with $f_3$ is getting larger from $r_m$ and so the resulting (added-material) blend surface is getting bigger, too. As $m_i$ decreases from 1, the blending range of $f_i$ with $f_3$ is getting smaller from $r_m$ to 0 so the resulting blend surface gets smaller. These can be found from the blend pointed by an arrow which is getting larger as $m_i$ increases in Fig. 7.

(5). When $B_U$ is used as a new primitive in another subtracted-material blend, such as $B_D(B_U(f_1,f_2), f_3)=1$ where the blend ranges for $B_U$ in $B_D$ are set $r_m$, varying $m_1$ and $m_2$ in $B_U$ enables primitives $f_1$ and $f_2$ to have different blending ranges

$$1-(1-r_m)^{m_i}=1, i=1 \text{ and } 2$$

to blend with $f_3$ in $B_D$.

As $m_i$ increases from 1, the blending range of $f_i$ with $f_3$ increases from $r_m$ and so the resulting (subtracted-material) blend surface shrinks more. As $m_i$ decrease from 1, the blending range of $f_i$ with $f_3$ decreases from $r_m$ and so the resulting blend shrinks less.

In addition, when $m_i=m_1$ and $2$, both are set 1, Equation (2) is similar to binary blends of the scale method. However, the major deference between $B_U(f_1,f_2)$ in (2) and $B_D(f_1,f_2)$ in (1) is that when $B_U$ is used as a new primitive in sequential blends, $m_1$ and $m_2$ in $B_U$ can be viewed as additional range parameters to adjust the subsequent added-material or subtracted-material blends of primitive $f_1$ and $f_2$ with other primitives, which solves the problem stated in the end of Section II.

B. Differentiable Blending Operators

To ensure that $B_U(x_1,x_2)$ in (2) is exactly a function, we need to find an arc-shaped curve $H(x_1,x_2)$ such that there exists a unique root $h_0$ of $T(h)=0$ for any $(x_1,x_2)$. In fact, many existing union blending operators on zero implicit surfaces satisfy this requirement and so can be used as a base curve in Step (1) in Section III.A, such as Hoffmann’s conic blend [12], Middleitch’s elliptic blends [13] and super-ellipsoidal blends [14]. Now, Hoffmann’s conic blend $H_{h}(x_1,x_2)$ is applied to develop $B_U(x_1,x_2)$ and it is represented by

$$H_{h}(x_1,x_2)=r_1 x_1^2 + r_2 x_2^2 + r_3 x_1^2 + r_4 x_2^2 + r_5 x_1^2 + r_6 x_2^2 + r_7 x_1^2 + r_8 x_2^2 + r_9 x_1 x_2 + 2p x_1 x_2$$
where \(-\theta \leq \varphi \leq \theta_{r2}\). Thus, \(H_{A2}(x_1, x_2) = 0\) in (2) can be given by \(H_{B2}(x_1, x_2) = 0\). Afterwards, the binary conic blending operator \(B_{U2}\) with range parameters \(r_1\) and \(r_2\) and curvature parameter \(p\) and parameters \(m_1\) and \(m_2\) to adjust the subsequent blends of \(f_1\) and \(f_2\) is given by:

\[
B_{A2}(x_1, x_2) = \begin{cases} 
  x_1^{1/m_1} x_2 \geq h_2 \\
  x_2^{1/m_2} x_1 \geq h_2 \\
  h_p \text{ otherwise } 
\end{cases} 
\]  

(3)

where \(h_1 = (1 + r_2) (x_1^{m_2/m_1})\), \(h_2 = (1 + r_1) (x_2^{m_1/m_2})\) and \(h_p\) is the root of the equation \(T(h) = 0\) for any \((x_1, x_2)\):

\[
T(h) = H_{B2}(x_1(h^{m_1})^{-1}, x_2(h^{m_2})^{-1}) \leq 0
\]

The root \(h_p\) of the equation \(T(h) = 0\) can be solved by Newton-Raphson numerical method and the value \(h = \text{Min}(x_1^{m_1}, x_2^{m_2})\) is the initial guess for solving \(h_p\).

C. Bulge Elimination

Similar to that in [8], [14], [15], union blend \(B_{U2}(f_1, f_2)\) from (2) or (3) is able to perform bulge elimination by replacing ranges \(r_1\) and \(r_2\) with positive position functions \(R_n\), \(n=1, 2\), to make the values of \(r_1\) and \(r_2\) approach 0 around the region \(\cos \theta = \pm 1\) by:

\[
R(x_1, x_2) = (G(x_1, x_2) + \omega)
\]

where \(G(x_1, x_2)\) maps \(R^2\) to \([0, 2]\) or \(R^2\) to \([0, 1]\), depending on which one of the following is used:

\[
G(x_1, x_2) = \begin{cases} 
  (1 - \cos \theta)^n \cos \theta \geq 0 \\
  0 \text{ otherwise } 
\end{cases} 
\]

(4)

(5)

And \(\theta\) is the angle between the gradients of \(f_1\) and \(f_2\) at the point \((x_1, x_2)\), \(n>1\) and \(\omega = 0\). \(G(x_1, x_2)\) in (4) maps to \([0, 2]\); \(G(x_1, x_2)\) in (5) maps to \([0, 1]\) which can be used to avoid unwanted blends or to avoid changing the topology of a union blend of closed primitives like those stated in [15], [16].

However, on bulge elimination, \(B_{U2}(f_1, f_2)\) along with \(R_i\) is almost similar to a pure union \(\text{Min}(f_1^{(lim1)}, f_2^{(lim2)})\) around the region \(\cos \theta = \pm 1\), where the value of \(G\) is close to zero. This means when \(m_2 \geq m_1\), \(\|V(f^{(lim)})\|\) might vary dramatically to \(\|V(f^{(lim)})\|\) around \(\cos \theta = 1\), and hence might cause computational errors in polygon-generation process. To reduce dramatic change in gradients, both the values of \(m_1\) and \(m_2\) are required to approach 1 around \(\cos \theta = 1\), by replacing parameters \(m_n\), \(n=1, 2\), with position functions \(M(x_1, x_2)\), which maps \(R_i\) to (1, \(m_i\)) for \(m_i > 1\) or (\(m_i, 1\)) for \(m_i < 1\), by:

\[
M = 1 + (m_i - 1)V(x_1, x_2)
\]

where \(V(x_1, x_2)\) maps to \([0, 1]\), given by:

\[
V(x_1, x_2) = \begin{cases} 
  (1 - \cos \theta)^n \cos \theta \geq 0 \\
  0 \text{ otherwise } 
\end{cases} 
\]

(4)

(5)

And \(\cos \theta\) is the same as in \(R_i(x_1, x_2)\) above and parameter \(n\) controls the rate \(M_i\) varies from 1 to \(m_i\) at.

IV. APPLICATIONS

\(B_{U2}(f_1, f_2)\) in (3) can be integrated with other existing blends such as those of the scale method and super-ellipsoidal blends. Its major applications are described below.

A. Application to an Added-Material Blend

When \(B_{U2}(f_1, f_2)\) is used as new primitives in an added-material blend, for example in \(B_{U2} (B_{U2}(f_1, f_2), f_3)\), changing \(m_1\) and \(m_2\) of \(B_{U2}\) can adjust the individual blend range of \(f_1\) and \(f_2\) with \(f_3\) as shown from the blend of \(f_2\) with \(f_1\) in Fig. 9. Fig. 10(a) displays two sets of four connecting cylinders defined by \(f_2 = B_{U2}(f_1, B_{U2}(f_1, f_3))\) and \(f_3 = B_{U2}(f_1, B_{U2}(f_2, f_3))\), respectively. The blend surfaces of union \(B_{U2}(f_n, f_3) = 1\) on the intersecting regions in Fig. 10(b) always have similar sizes because of defining \(f_n\) and \(f_3\) using \(B_{A2}\) in (1). However, the three surfaces from left to right in Fig. 10(c) indicates that if \(f_n = B_{U2}(f_1, B_{U2}(f_2, f_3))\) and \(f_3 = B_{U2}(f_1, B_{U2}(f_2, f_3))\) defined by (3) are used instead, then union \(B_{U2}(f_n, f_3)\) can individually adjust the sizes of both the blend surfaces on the intersecting regions by setting both \(m_1\) for \(f_2\) in \(f_n\) and \(f_3\) in \(f_2\) to 0.2, 1.2 and 1.8, respectively.

B. Application to a Subtracted-Material Blend

When \(B_{U2}(f_1, f_2)\) is used as a subtracting or a subtracted operand in a difference blend, for example \(B_{D2} (B_{U2}(f_1, f_2), B_{U2}(f_1, f_3))\), changing \(m_1\) and \(m_2\) of \(B_{U2}(f_1, f_2)\) can adjust the individual blend range (or the size) of the subtracted-material blends of \(f_1\) and \(f_2\) from \(B_{U2}(f_1, f_3)\). This can be seen in Fig. 11, which displays a difference blend of bigger intersecting cylinders \(B_{D2}(f_1, f_3)\) in Fig. 11(a) from another one \(B_{U2}(f_1, f_3)\) in Fig. 11(b), defined by \(B_{D2}(B_{U2}(f_1, f_2), B_{U2}(f_1, f_3))\) and \(B_{U2}(f_1, f_3)\) in Fig. 11(d). Fig. 11(c) shows the same blend as Fig. 11(d) does except \(B_{A2}\) is used instead of \(B_{U2}\) and so all the edges on it always have similar sizes.
range parameters behave like pure union or intersection, blending surfaces in sequential blends. In order to solve their primitives always have similar sizes of subsequent method that is able to transform an existing union blend the problem, this paper has proposed a generalized into a new binary union blend that

\[ B(f_1, f_2, f_3) \]

where \( f_1 \) and \( f_3 \) are specified by arrows, are thinner than the others. (d) The same difference as in (c) except that \( m_1 = 1 \) and \( m_3 = 1 \) and hence all the edges have similar sizes.

V. CONCLUSION

In Ricci’s constructive geometry, existing blends with range parameters behave like pure union or intersection, Max or Min, in non-blending regions. This implies that their primitives always have similar sizes of subsequent blending surfaces in sequential blends. In order to solve the problem, this paper has proposed a generalized method that is able to transform an existing union blend into a new binary union blend that

- Provide primitives additional parameters \( m_1 \) and \( m_2 \) to individually adjust the blending range and the size of each primitive’s subsequent blend in sequential blends, and especially the blend surface does not change no matter what values the parameters are set.
- Provide primitives range parameters to adjust the size of the transition of each primitive’s blend surface and to make the transition generated in a specified region such that primitives deform locally.
- Are \( C^1 \) continuous everywhere and so can be used as a new primitive in other blends to generate sequential blends with overlapping blending regions.

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REFERENCES


Pi-Chung Hsu was born in 1966. He received his doctor degree in information engineering at national Sun Yat-sen University, Taiwan in 2003. Now he is an associate professor at Shu-Te University, Taiwan. His research directions include 3D computer graphics, implicit surfaces and solid modeling.