Hierarchy Construction of Mathematical Knowledge

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***Abstract*—Visualization with a hierarchy of mathematical knowledge structure, assist learners in understanding mathematics. We are developing a method which constructs a hierarchy of mathematical knowledge by using hyperlink analysis in our mathematics e-learning site. Hierarchy is calculated by an adjacency matrix.**

***Index Terms*—hierarchy construction, graph, adjacency matrix, mathematical knowledge, node, edge**

I. Introduction

Recently E-learning becomes widely used not only in education institutions and companies, but also on a personal basis. There are many types of e-learning and many ways to categorize it as well known [1]. We categorized it into two types. The one is the type which has a course instructional designed by educators, and the other is the type in which learner tries to make their own learning path by using many electric learning materials. Massive Open Online Courses (MOOC) [2], [3] like Coursera, Udacity and edX which is of particular interest nowadays is categorized into the former type. Learning using search engines categorized into the latter type. Many people use search engines in studying on a daily basis. Despite that, there are few researches about the latter type.[4]-[6] We focus on the latter type of e-learning and have developed Japanese mathematics e-learning site, "KIT Mathematics Navigation [7]", with over a thousand pages in order to assist the learning using search engines [8]. The concept of our site is "Online Mathematics Reference". Visitors who access our site through search engines use our mathematical materials by clicking the links to see the reference pages. In other words, they make their own learning path by selecting the links. We call this learning style "Link Back Learning" [8].

On the other hand, mathematics has a complicated hierarchical knowledge structure. In studying mathematics, learners usually learn it step by step from low level to high level in the hierarchy of mathematical knowledge. We can see some parts of its hierarchy on the contents page of mathematics books, and so on. But it is difficult to view the whole mathematical knowledge hierarchy.

Manuscript received June 9, 2014; revised August 13, 2014.

Recently graph visualization methods attract attention from various fields. We have been developing the visualization of mathematical knowledge structure from link structure of the mathematics e-learning site, "KIT Mathematics Navigation", which is written in Japanese [9]. Its graph visualization is open to the public on the Internet [10]. We think that this visualization is very useful to grasp mathematical knowledge structure and make learning mathematics more efficient. Now we are going to construct a hierarchy of mathematical knowledge by using link structure data of our e-learning site. We think that this hierarchical graph visualization of mathematical knowledge structure will assist learners in making their own learning path.

II. Visualization of Mathematical Knowledge Structure

A. Mathematics E-learning Site

E-learning site "KIT Mathematics Navigation" is open to the public on the Internet. On a weekday in May in 2014, about 8 thousand people a day access our site. About 80% of traffic source are search engines according to google analytics.

Our site is translated by Google Translate from Japanese to many languages (i.e., 81 languages). So far, the quality of translation is not enough [7]. Fig. 1 shows the sample of the web page.

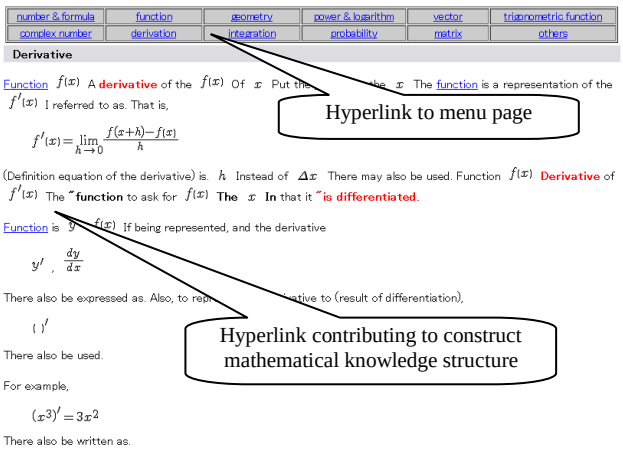

Figure 1. Screen shot of a webpage on our e-learning site which is translated by Google Translate

Our site consists of two parts. One is the reference part and the other is the exercise part. Each web page in

doi: 10.12720/lnit.2.2.203-207

reference part has one unit mathematical knowledge and links are set on keywords in contents so as to refer to other web pages. As a result the directions of the links are from higher level to a lower level in the hierarchy of mathematical knowledge. These links create relationships among units of mathematical knowledge, i.e., mathematical knowledge structure. The link structure of our site makes it possible to study by "Link Back Learning" method [8]. Fig. 2 illustrate the "Link Back Learning" method in contradistinction to step by step learning.

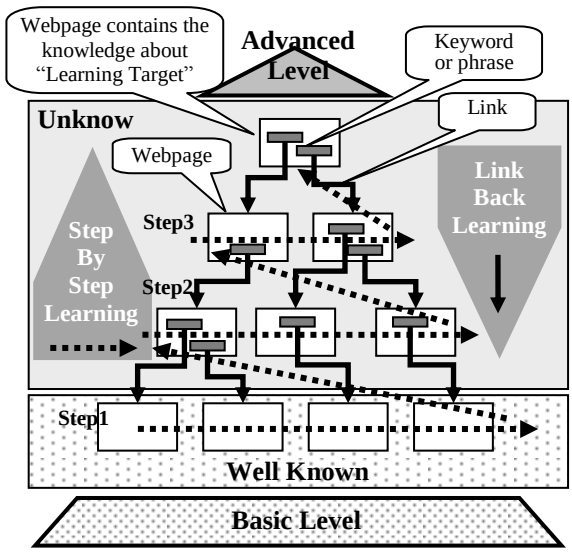

Figure 2. Illustration of link back learning

B. Graph Visualization

We developed a crawler program with PHP and MySQL to analyze the network structure of our site. We eliminate extra data which do not consist of knowledge structure such as menu pages and links connecting to menu pages. We use the gephi [11] which is open source software to visualize the knowledge structure. We display a graph drawing on a web page by using the JavaScript GEXF Viewer for Gephi under MIT License [12]. Fig. 3 shows the directed graph drawing of the mathematical knowledge structure. Nodes of graph mean web pages which shown by circles. Edges of graph mean hyperlinks which have directions. The size of the nodes (circle) is depending on the number of inbound links. Large nodes means to have many inbound links or key knowledge of mathematics. Node labels are titles of web pages which are translated Japanese to English by Google Translate. The number of nodes is 791. GEXF Viewer has several functions. If we select a node, a side menu appears and we can see the list of titles of web pages connected by inbound links and the list of titles of web page connected by outbound links as shown Fig. 4. Edges are shown by circle arcs. The directions of the edge are set in clockwise direction. We can move on from outbound node to inbound node along the line of arc in a clockwise fashion. This function is very useful to learn the basic or advanced knowledge relative to the knowledge written in a selected node (web page). We can study in the direction of advanced mathematical knowledge efficiently while we grasp the mathematical knowledge structure. We added a function to GEXF Viewer for Gephi by modifying the javascript source. We set hyperlink on top of side menu to access the selected web page on the graph drawing. This hyperlink combines with Graph drawing and our web site seamlessly.

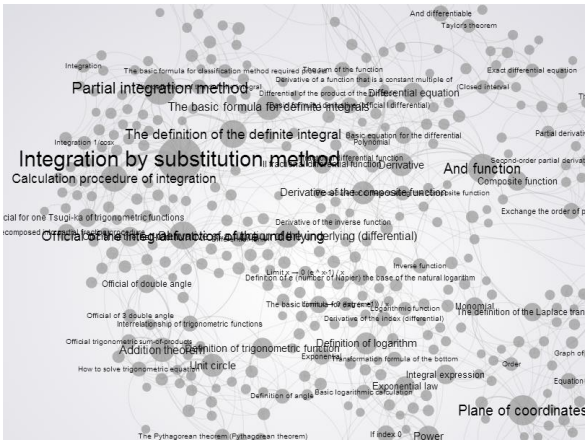

Figure 3. Screen shot of directed graph drawing of mathematical knowledge produced by GEFX Viewer for Gephi

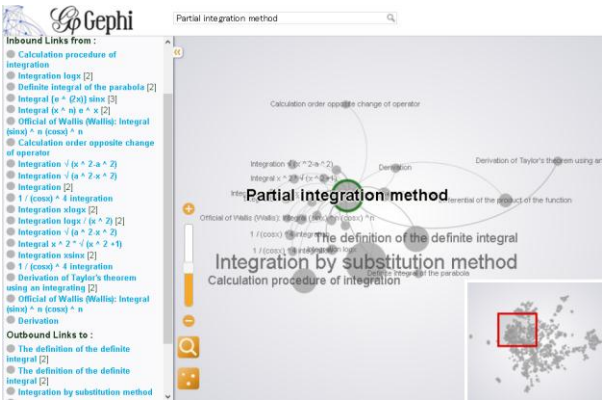

Figure 4. Screen shot of selected node

III. Idea of Hierarchy Construction

We create link structure, i.e., directed graph of small mathematics e-learning site as shown in Fig. 5 in order to explain the hierarchy construction. There is a hierarchy among web pages as mentioned. Web page 6 has hyperlink which connects to web page 1 as a reference. Basically hierarchy of mathematical knowledge in web page 6 is higher than that in web page 1. Table I shows the adjacency matrix of graph in Fig. 5. The component (1, 4) =1 means the hyperlink which coming from web page 0 and then going to web page 3.

We are planning to list web pages in higher hierarchical order under the hypothesis that higher hierarchal mathematical knowledge tend to refer to higher hierarchical mathematical knowledge than lower one. As shown in Table I, web page 0 has two outbound hyperlinks to web page 3 and 8 and web pages 1 has two outbound hyperlinks to web page 2 and 8. We consider that mathematical knowledge in web page 1 is a higher

hierarchy than that in web page 0. Then we swap row 1 and row 2 and swap column 1 and column 2 from adjacency matrix as shown in Table II. We swap two adjacent rows and two adjacent columns in ascending order of row number until adjacency matrix becomes the state of Table III. The order of web page number in row and column indicates the hierarchy of web page shown in Fig. 5. Fig. 6 show the same graph, but shows the hierarchy by vertical position. The page 4, 6 and 7 are high level of hierarchy and the page 5 and 8 are low level of hierarchy.

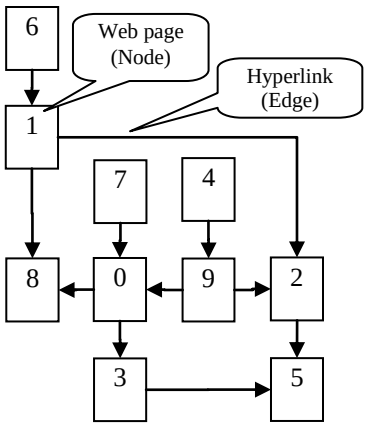

Figure 5. Example of graph of small mathematics e-learning site

TABLE I. ADJACENCY MATRIX OF FIG.5

		Web page number (inbound side)									
		0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Web page number (outbound Side)	**0**				1					1	
	1			1						1	
	2						1				
	3						1				
	4										1
	5										
	6		1								
	7	1									
	8										
	9	1		1							

TABLE II. ADJACENCY MATRIX AFTER SWAPPING ROW 1 AND ROW 2 AND THEN SWAPPING COLUMN 1 AND COLUMN 2 FROM ADJACENCY MATRIX OF FIG.3

		Web page number (inbound side)									
		1	**0**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Web page number (outbound Side)	**1**			1						1	
	0				1					1	
	2						1				
	3						1				
	4										1
	5										
	6	1									
	7		1								
	8										
	9		1	1							

TABLE III. ADJACENCY MATRIX AFTER HIERARCHY CALCULATION

		Web page number (inbound side)									
		4	**6**	**7**	**9**	**1**	**0**	**2**	**3**	**5**	**8**
Web page number (outbound Side)	**4**				1						
	6					1					
	7						1				
	9						1	1			
	1							1			1
	0								1		1
	2									1	
	3									1	
	5										
	8										

IV. CALCULATION OF HIERARCHY CONSTRUCTION

Now we are developing a program which orders web pages in higher hierarchical order by using PHP and MySQL. Firstly, the program calculates the 791-by-791 adjacency matrix data from the data of graph visualization of mathematical knowledge structure. We set the data table in MySQL as shown in table IV in order to express the adjacency matrix. Each datum in row indicates the each component in adjacency matrix. In this case, the row number starts from number 0. The initial order of node id of row and column is the order of crawling our site. So row number and outbound web page number are same value and column number and inbound web page number is the same value for each hyperlink number. The web page 0 has two outbound links to web page 6 and 13. Secondly, the program compares with the data of the adjacency rows in ascending order of row number as noted before. Table V shows the result after the hierarchy construction between Row 0 and Row 250. Rows between 251 and 790 have not been listed in hierarchy order. So the column number and inbound webpage number for hyperlink number 3 are still same value, i.e., calculating hierarchy does not reach row number 323.

We have some troubles that some swapping repeat infinitely. We found that there are some graph structures as shown in Fig. 7. Most of hyperlinks outbound from web page of higher level of hierarchy and inbound to that of lower level hierarchy. There are some hyperlinks which outbound from a lower level and inbound to a higher level so as to learn more effectively and efficiently. We check the web pages related to infinite repeat and delete the data of hyperlink which outbound from a lower level and inbound to higher level manually.

Table VI shows the portion of a hierarchy of mathematical knowledge after calculating until row number 713. The hierarchy of Integration is higher than that of differentiation. The some part of calculated hierarchy is unreasonable. For example the knowledge of factor is comparatively higher level in calculated hierarchy. We have to improve link structure so that calculated hierarchy become more reasonable.

According to the increase of the webpage, it takes an awful long time to accomplish hierarchy construction in this program that swap two adjacent rows in ascending

order of row numbers. As we have been developing our e-learning site, the number of web wage will increase in the future. We have to construct a hierarchy of mathematical knowledge on a periodic basis. We will use the previous hierarchy data as an initial order of rows in order reduce calculation time from the next calculation of hierarchy.

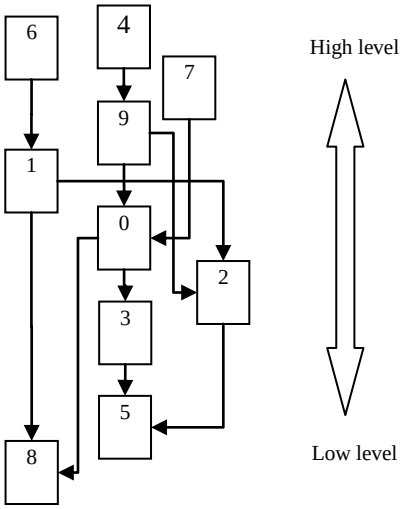

Figure 6. Same graph of Fig. 2 with hierarchy

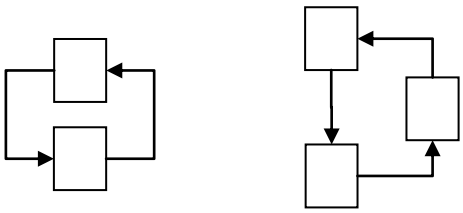

Figure 7. Example of graph structure causing troubles

TABLE IV. DATA TABLE OF ADJACENCY MATRIX IN INITIAL STAGE

Hyper link Number	Row Number	Outbound Web Page Number	Column Number	Inbound Web Page Number
0	0	0	6	6
1	0	0	13	13
2	1	1	9	9
3	1	1	323	323
:	:	:	:	:
:	:	:	:	:

TABLE V. SAMPLE OF DATA TABLE OF ADJACENCY MATRIX AFTER HIERARCHY CONSTRUCTION BETWEEN ROW 0 AND ROW 250

Hyper link Number	Row Number	Outbound Web Page Number	Column Number	Inbound Web Page Number
0	100	0	151	3
1	100	0	127	13
2	61	1	77	9
3	61	1	323	323
:	:	:	:	:
:	:	:	:	:

TABLE VI. PORTION OF HIERARCHY OF MATHEMATICAL KNOWLEDGE AFTER CALCULATING UNTIL ROW NUMBER 713.

0	Calculation Procedure of Integration
1	Convenient Formula for Integration
2	Integration by substitution ax+b=1
3	Integration, High Degree Trigonometric Functions (2)
4	Integration, High Degree Trigonometric Functions (1)
:	
100	Common Multiple
101	Factor Theorem
102	Procedure for resolution into factors
103	Formula for resolution into factors
104	Unit Vector Notation
:	
200	Product to Sum/Difference Formulas in Trigonometric Functions
201	Integration, 1/(sin x) ^4
202	Integration, 1/(cos x)^4
203	Formula for Degree Lowering in Trigonometric Functions
204	Solution of x^3=1
300	Calculation Example 2 of Degree Lowering
301	Calculation Example 2 of Degree Lowering
302	Regular Matrix
303	Common Logarithm
304	Relationship of Trigonometric Ratio and the Length of Each Side of a Right Triangle
:	
400	Derivation Procedure for Definition of Determinant
401	Linear Combination
402	Exponent Function
403	Example of Exponent Function
404	Unit Circle
:	
500	Event
501	Series
502	Even
503	Odd
504	Irrational
:	

V. EFFECT AND FUTURE WORK

We have a plan to add hierarchy information to graph visualization by coloring node circles. For example circles are colored from blue to red gradually according to hierarchy. We think that graph visualization of mathematical knowledge structure with hierarchy will help learners learn mathematics more efficiently as compared with the previous one as shown in Fig.3.

We will check the result of hierarchy construction whether hierarchy of mathematics knowledge is reasonable or not. We will modify and improve our site as need arises.

We can make the system which produces hierarchy trees of mathematical knowledge which starts selected mathematical knowledge based on the data of hierarchy

of mathematical knowledge. This system helps us to understand what the selected knowledge is based on. We can also make the system which produces reverse hierarchy trees of mathematical knowledge to show us what the selected knowledge are applied to.

We have also planned to combine access log data with hierarchical data to analyze the learning path which learners make at our site. This analysis will reveal whether learners tend to learn advanced knowledge or basic knowledge.

ACKNOWLEDGMENT

This work was supported by JSPS KAKENHI Grant Number 25350359. We appreciate the student staff who engaged in developing our e-learning site.

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Akira Nakamura was born in Japan in 1961 and received his Master of Engineering degree from Osaka University in 1986. He joined the Matsushita Electric Industrial Co., Ltd. (Panasonic) in 1986 and was engaged in the research and development of the liquid crystal display. He joined the Kanazawa Institute of Technology in 2004 and presently holds the rank of professor. He has been engaged in teaching mathematics and physics, and in the research and development of e-learning.

