Power Control Algorithms for CDMA over Wireless Fading Channels

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***Abstract*—The power control subject; is essential in mobile communication systems, because it can mitigate the near-far problem, increases the system capacity, improves the quality of service, increases the battery life of the mobile terminal, and decreases the biological effects of electromagnetic radiation. In this paper, three different types of power control algorithms will be studied which are; Distributed Power Control (DPC) algorithm, Distributed Balanced Algorithm (DBA), and Foschini's and Miljanic's Algorithm (FMA), the Matlab code has been built for each of the three listed algorithm and the results have been presented and finally the comparison between the performance of each of the three algorithms has been generated.**

***Key Words*—power control, CDMA, FDMA, TDMA, quality of service, capacity improvement near far effect.**

I. Introduction

Direct Sequence Code Division Multiple Access (DS/CDMA) is an emerging technology for civilian wireless communication systems. CDMA offers improved performance in terms of capacity or coverage area over FDMA or TDMA based cellular networks, another advantage is that CDMA combats multipath fading due to the fact that the signal is spread over a large bandwidth, and that each path can be tracked separately at the receiver end and no frequency or time management is required.

In the reverse link, where several asynchronous users are communicating with the same base station and share the same bandwidth, inter-user interference due to non-zero crosscorrelation between different codes adds up in power. The quality of each individual link degrades as the number of users in the system increases

In FDMA and TDMA the number of available frequencies and time slots are the limiting factors for the number of users. Blocking occurs when the number of users exceeds the available frequencies and time slots, where as in CDMA blocking occurs when the tolerance limit to interference is exceeded. Therefore in CDMA the level of interference is the limiting factor.

In order to meet the increasing demand of mobile subscribers for various services such as multimedia, internet, transferring of big data like digital pictures, it is crucial to have higher capacity and more severe Quality of Service (QoS) requirement, to meet this requirements new technologies and improved resource management including channel assignment, power control and handoff are needed.[1], [2].

II. Near Far Problem

Consider that there are 2 mobile stations (MS) transmitting at equal powers, but one is nearer to the base station (BS) compared to the other as shown in Fig. 1. The BS will receive more power from the nearer MS and this makes the farther MS difficult to understand. As we know, the signal of one MS is the noise for another MS and vice-versa. So the Signal-to-noise interference ratio (SINR) for the farther MS is much lower. If the nearer MS transmits a signal that is orders of magnitude higher than the farther MS then the SINR for farther MS may be below detectability threshold and it would seem that the farther MS is not at all transmitting. This situation is called "near-far problem" and is less pronounced in GSM than CDMA-based systems as the MS transmit at different frequencies and timeslots in case of GSM. To overcome this problem, a power control mechanism is used so as closer MSs are commanded to use less power so that the SNR for all MSs at the BS is roughly the same [3], [4]

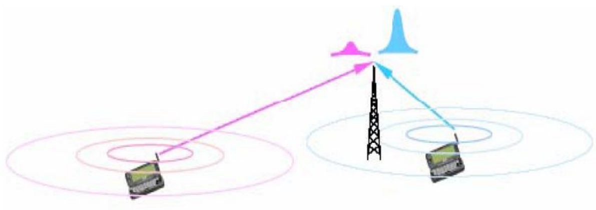

Figure 1. Near far effect

III. Power Control

Power control is a necessary element in all mobile systems because of the battery life problem and safety reasons, but in CDMA systems, power control is essential because of the interference-limited nature of CDMA. The overall objectives of power control can be summarized as follows:

1) Overcoming the near-far effect in the uplink
2) Optimizing system capacity by controlling interference

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3) Maximizing the battery life of mobile terminals

Power control in CDMA is a closed-loop PC which is a combination of outer and inner closed loop control as shown in Fig. 2. The inner (also called fast) closed loop PC adjusts the transmitted power in order to keep the received Signal-to-noise Interference Ratio (SINR) equal to a given target.

This SINR target is fixed according to the received BLER (Block Error Rate) or BER (Bit Error Rate). The setting of the SINR target is done by the outer loop PC, which is part the Radio Resource Control Layer, in order to match the required BLER. Outer loop PC update frequency is 10-100 Hz. The BLER target is a function of the service that is carried. Ensuring that the lowest possible SINR target is used results in greater network capacity [3], [4]

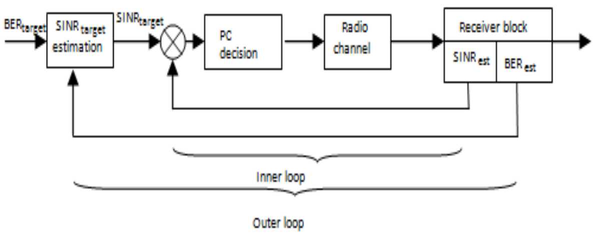

Figure 2. Power control in CDMA system. In the receiver block, the received SIR and BER are estimated and used respectively for the inner-loop and the outer-loop.

IV. POWER CONTROL ALGORITHMS

Power control is essential in mobile communication systems, because it can mitigate the near-far problem, increase the system capacity, improve the quality of service, increase the battery life of the mobile terminal, and decrease the biological effects of electromagnetic radiation.

The objective of the power control algorithm is to keep the transmitted power (for the mobile station in the uplink power control and for base-station in downlink power control) at the minimum power required to achieve the target Quality of Service (QoS) in the system[5], [6].

Before giving a precise mathematical formulation for the optimum power control problem, some notations and definitions are given. Let the *transmitted power control* vector be a Q-dimensional column vector $\mathbf{P} = [P1, P2... PQ]'$, where Pi is the transmitted power of user i. SINR of user I is denoted by Γ_i.

Mathematically the power control problem is formulated as follows:

The power control vector **P** that minimizes the cost function

$$J(P) = 1P = \sum_{i=1}^{Q} P_i \qquad (1)$$

$$\Gamma_{ki} = \frac{P_i G_{ki}}{\sum_{\substack{j=1\\ j\neq i}}^{Q} P_j G_{kj} + N_i} \geq \Gamma_{\min}, \qquad (2)$$

$$I = 1\ldots..,Q,\ k=1\ldots\ldots M.$$

$$P_{\min} \leq P_i \leq P_{\max},\ \forall\ i=1,\ldots\ldots.,Q. \qquad (3)$$

where:
1= [1, 1, 1……].
Q: Number of mobile stations.
M: Number of base stations.
G_{kj}: Channel gain between mobile station j and base station k.
N_i: The average power of the additive noise at receiver i.
P_{max}: Maximum power, which can be handled by the transmitter.
P_{min}: Minimum power, which can be handled by the transmitter.
Γ_{min}: Minimum predefined SINR

For simplicity, we will refer to user I without using the subscript of its assigned base station. For example, we will use Γ_i instead of Γ_{ki}. If the SIN of user I, $\Gamma_i < \Gamma_{min}$, and the transmitted power $Pi = P_{max}$, then user I (or some other users) has to be dropped from this link. Another important factor is the target SINR (Γ^T). It should be noted that the superscript (T) means (Target). The dash (′) is used to indicate transpose operation. The difference between the target SINR and the minimum predefined SINR is called SINR margin.

The target SINR value is determined by the outer-loop power control to achieve certain QoS in the cell. The target SINR could be different from user to user because it depends on the type of service requested by the user.

A. Centralized Power Control

If the information of the link gains and the noise levels are available for all users, then the centralized power control algorithm can be applied to solve the power control problem given in (1),(2) . For noiseless case, $N_i = 0$, (2) becomes

$$\Gamma_i = \frac{P_i G_{ki}}{\sum_{\substack{j=1\\ j\neq i}}^{Q} P_j G_{kj}} \geq \Gamma_{\min}, \qquad (4)$$

$$I = 1\ldots..\ Q,\ k=1\ldots\ldots M$$

Equation (4) can be written in a matrix form as

$$P \geq \Gamma_{\min} HP. \qquad (5)$$

where **H** is a nonnegative matrix with the following elements

$$(H)_{ij} = \begin{cases} 0 & i=j. \\ \frac{G_{kj}}{G_{ij}} & i\neq j. \end{cases} \qquad (6)$$

The problem is how to find the power vector P > 0 such that (5) is satisfied. Equation (5) can be written as

$$\left[\frac{1}{\Gamma_{\min}} I - H\right] P \qquad (7)$$

The inequality is dropped in (7), since equality sign holds for the minimum power vector. It is known from linear algebra that a nontrivial solution of (7) exists if and only if [$\frac{1}{\Gamma_{\min}} I - H$] is a singular matrix. It is seen from (7) that this happens, if $\frac{1}{\Gamma_{\min}}$ is an eigenvalue of **H,** and the optimum power vector **P** is the corresponding eigenvector. The power vector **P** should be positive. Perron-Frobenius theorem [7] says that for a nonnegative and irreducible QxQ matrix **H** there exists a positive vector **P** associated with the maximum eigenvalue.

$$\Lambda^* = \rho(\mathrm{H}) = \max |\lambda_i| , \; i = 1, \ldots\ldots, Q \,. \tag{8}$$

where λi is the i^{th} eigenvalue of the matrix **H**, and ρ(**H**) is the *spectral radius* of matrix **H.**

Based on this the maximum achievable SINR can be expressed as

$$\gamma^* = \frac{1}{\lambda^*} = \frac{1}{\rho(H)} \,. \tag{9}$$

Now by considering the additive white noise at the receivers, (2) can be written in a matrix form as

$$[\,\mathrm{I}\text{-}\Gamma^T H\,]\,\mathrm{P} \geq \mathrm{u}. \tag{10}$$

where **u** is a vector with positive elements u_i specified by

$$\mathrm{u}_i = \frac{\Gamma^T N_i}{G_{ki}} \,, \quad i = 1, \ldots, Q, \; k = 1, \ldots, M. \tag{11}$$

It can be shown [7] that if $\Gamma^T < \frac{1}{\rho(H)}$ then the matrix [I- Γ^TH] is invertible and positive. In this case, the power vector **P*** is

$$\mathrm{P}^* = [\,\mathrm{I}\text{-}\Gamma^T H\,]^{-1}\,\mathrm{u}. \tag{12}$$

The solution of the optimization problem posed in (1), (3). [8].

The computation of the optimum power vector using the centralized power control algorithm needs the link gains of all users. This is computationally intensive; moreover it is not feasible particularly in multi-cell cases. Therefore it is common in practice to use a distributed power control technique. Centralized power control can be applied to test the upper bound performance using a distributed technique in simulation.

B. Distributed Balancing Algorithm (DBA)

Zander has proposed a Distributed Balancing Algorithm [8]. The method is based on the power method for finding the dominant eigenvalue (spectral radius) and its corresponding eigenvector.

The DBA algorithm is as follows

$$\mathrm{P}_i(t+1) = \beta\, \mathrm{P}_i(t) \left[1 + \frac{1}{\Gamma_i(t)}\right]. \tag{13}$$

$$\beta > 0, \; t = 0,1,\ldots \quad i = 1,\ldots\ldots,Q$$

$$\mathrm{P}(0) = \mathrm{P}_0 \quad \mathrm{P}_0 > 0$$

The algorithm starts with an arbitrary positive vector **P** (0). The SINR level $\Gamma_i(t)$ is measured in link i. If the power control is for downlink, then the measurement of the SINR is made at the mobile. The result is to be reported back to the base station. The transmitter power at the base station is then adjusted according to the DBA in (13). If the power control is for uplink, then the measurement of the SINR has to be made at the base station.

The result has to be reported back to the mobile, and each mobile station will adjust its transmitted power according to the DBA.

- *Proposition (1)*

Using the DBA algorithm (13) the system will converge to SINR balance with probability one, i.e.,

$$\lim_{n\to\infty} P(t) = P^* \quad t = 0,1,\ldots$$

$$\lim_{n\to\infty} \Gamma_i(t) = r^* \quad i = 1,\ldots,Q \tag{14}$$

where γ* is the maximum achievable SINR, which is equal to 1/λ*. As before, λ* is the spectral radius of the nonnegative matrix **H**, and **P*** is the corresponding eigenvector representing the optimum transmitted power.

Proof: See [8]

It is clear that the DBA uses only local SINR information and utilizes an iterative scheme to control the transmitted power. The main disadvantage of the DBA is that its convergence speed is not satisfactory. If the allowed speed of the iterations is not high enough, then the distributed algorithm may result in an outage probability much greater than the optimum value [9].

A. The Distributed Power Control (DPC)

It has been shown that the distributed power control scheme for satellite systems can be applied to cellular systems [10]. The results presented in [10] indicate that the DPC scheme has the potential to converge faster than the DBA scheme at high SINR's.

The power adjustment made by the i^{th} mobile at the t^{th} time slot is given by

$$\mathrm{P}_i(t+1) = \beta(t)\, \frac{P_i(t)}{\Gamma_i(t)} \,, \quad i = 1,\ldots\ldots,Q \,, \; t = 0,1,\ldots\ldots \tag{15}$$

where β(t) is some positive coefficient chosen to achieve the proper power control vector (not too large or too small). In additive noise environment, it is very common to select β (t)=Γ^T [5].

C. Foschini's and Miljanic's Algorithm (FMA)

Foschini and Miljanic have proposed a simple and efficient distributed power control algorithm [11]. The proposed algorithm is based on the following continuous time differential equation:

$$\Gamma_i(\tau) = -\beta\,[\,\Gamma_i(\tau) - \Gamma^T\,],\; \beta > 0,\; \tau \geq 0. \tag{16}$$

The steady state solution of the above differential equation for user i is $\Gamma_i = \Gamma^{\mathrm{T}}$.

The speed of the convergence depends on the coefficient β.

Define the *total interference of user i*:

$$\mathrm{I}_i(\tau) = \sum_{j\neq i}^{Q} G_{kj}(\tau) P_j(\tau) + N(\tau) \tag{17}$$

Hen Γ_i from (2) becomes

$$\Gamma_i(\tau) = \frac{G_{ki}(\tau) P_i(\tau)}{\sum_{j\neq i}^{Q} G_{kj}(\tau) P_j(\tau) + N(\tau)} = \frac{G_{ki}(\tau) P_i(\tau)}{I_i(\tau)},$$

$$i=1\ldots Q,\ k=1\ldots M. \qquad (18)$$

Assuming that $I_i(\tau)$ and $G_{ki}(\tau)$ are constant, substituting (18) into (16) gives

$$\frac{G_{ki}P_i(\tau)}{I_i} = -\beta\left[\frac{G_{ki}P_i(\tau)}{I_i} - \Gamma^T\right] \qquad (19)$$

$$i=1,\ldots,Q,\ k=1,\ldots,M.$$

Using (17) becomes

$$P_i(\tau) = -\beta\left[P_i(\tau) - \frac{\Gamma^I}{G_{ki}}\left(\sum\nolimits_{i\neq j}^{Q} G_{kj}(\tau)P_j(\tau)+N\right)\right] \qquad (20)$$

$$i=1,\ldots,Q,\ k=1,\ldots,M.$$

Using matrix notation one can write (20) as

$$P(\tau) = -\beta[\,I-\Gamma^T H]\,P(\tau) + \beta\,u. \qquad (21)$$

At the steady state, we have

$$P^* = [I-\Gamma^T H]^{-1}\,u. \qquad (22)$$

- *Proposition (5)*

If there is a power vector **P*** , for which the target Γ^T values are attained, then no matter what is the initial P_i (0), each of the P_i (τ) evolving according to (19) will converge to **P*** of (22).
Proof: see [11].

The discrete form of (22) is

$$P(t+1) = \beta\left[\left(\frac{1}{\beta}-1\right)I + \Gamma^T H\right]P(t) + \beta u,\ t=0,1,\ldots \qquad (23)$$

and the iterative power control for each user i is

$$P_i(t+1) = (1-\beta)\,P_i(t)\left[1+\frac{\beta}{(1-\beta)}\left(\frac{\Gamma^T}{\Gamma_i(t)}\right)\right], \qquad (24)$$

$$t=0,1,2,\ldots,\ i=1,\ldots,Q.$$

V. Simulation Environment

To show the effectiveness of the proposed algorithms, the scenario assumed 100 users uniformly distributed in an area of 4 km^2 with four base stations. Perfect handover is assumed where each user is assigned to the base station with the least gain loss. The channel is assumed static. The simulation code assumes the radio channel with propagation loss and shadowing. The received signal power at base station [j] due to user [i] is assumed to follow power law:

$$\hat{P}_{ji} = P_i\frac{10^{S_{ji}}}{d_{ji\alpha}}. \qquad (25)$$

where S_{ji} is the shadowing variable in the path from i-th mobile station to j-th base station and it is assumed to be a random variable with log-normal distribution and 6 dB variance, d_{ji} is the distance between user i and base station j, α=4. And the average additive noise in Watts at the input of the base station is $N=KT_oN_FB$, Where: K=1.38*10^{-23} (Poltizman Constant), T_o =290 (K-Temperature), N_F=2dB (noise Figure), B=5MHz (Bandwidth).

VI. Simulation Results

The simulation result figures of the studied Distributed Balancing Algorithm, Distributed Power Control and Foschini's and Miljanic's Algorithm (FMA) for the above described scenario are; Fig. 3, 5, 7 show the Transmit power of different chosen users with respect to the time iteration at -15 dB SINR , and Fig. 4, 6, 8 show the SINR of the same users.

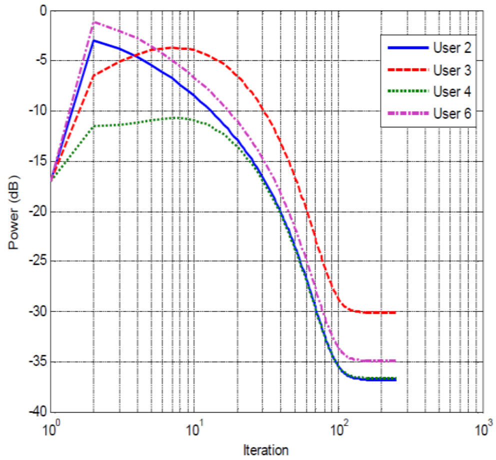

Figure 3. Transmit power with time iteration using DBA algorithm at -15 dB threshold SINR.

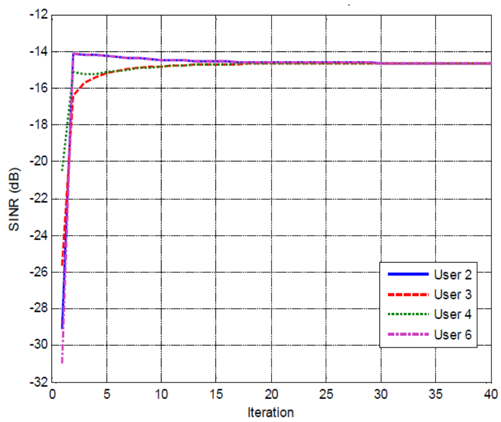

Figure 4. The achieved SINR for 4th terminal at -15 dB threshold SINR

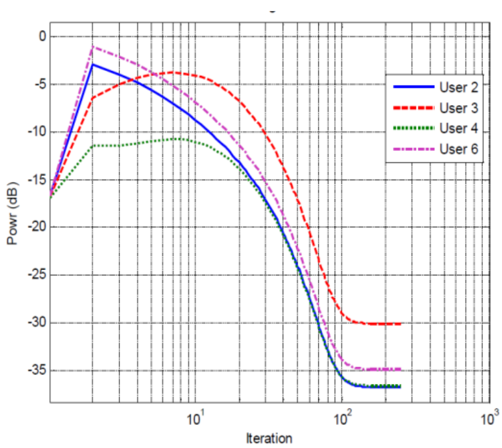

Figure 5. Transmit power with time iteration using DPC algorithm at -15 dB threshold SINR

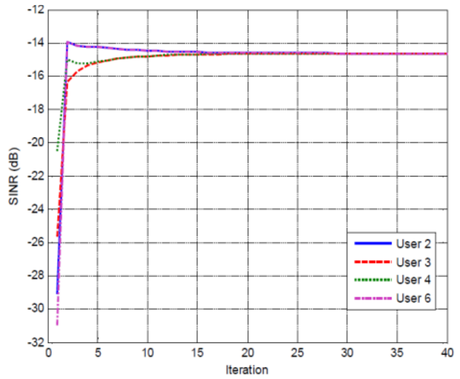

Figure 6. The achieved SINR for 4th terminal at -15 dB threshold SINR

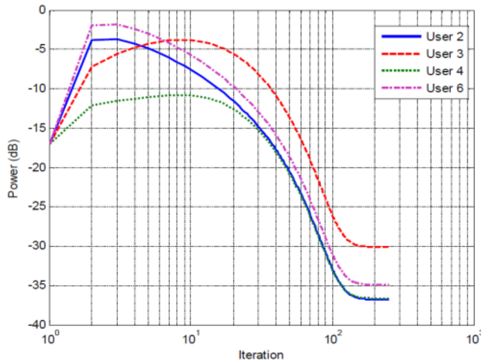

Figure 7. Transmit power with time iteration using FMA algorithm at -15 dB threshold SINR

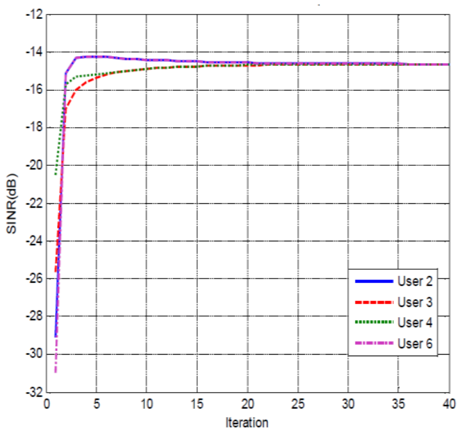

Figure 8. The achieved SINR for 4th terminal at -15 dB threshold SINR

VII. COMPARISON of THE THREE STUDIED ALGORITHMS

Fig. 9, shows the compression between the three studied algorithms which are; DPC (—— curve), DBA (- - - -curve), and FMA (………. curve). From this figure we can decide that the order of the base algorithms on the speed and reaching to the optimum value of the power are; DPC, DBA and FMA. Where all of the three algorithms were reaching to the optimum power value, which is the estimated base on CPC algorithm.

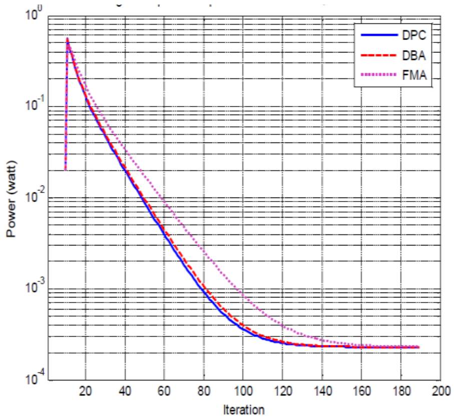

Figure 9. Convergence speed comparison between DPC,DBA and FMA

VIII. CONCLUSION

In this paper the comparison between the performance of each of the three algorithms has been generated, thus the DPC was the fastest over the other two algorithms and the DBA was faster than FMA, considering the time iteration slot to reach to the optimum power value.

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