Reliability and Profit Analysis of a Soft Hot Standby PUSH VOD System based on Markov Model

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Abstract—Based on Markov model, this paper analyzes a soft standby's reliability and safety in PUSH system for digital TV, which is constituted by two PUSH VOD severs working on master-slave mode. It illustrates 5 interesting rules that can guide us to design a more reliable and more secure system. We will see some key parameters that is positive correlated with the reliability may be negative correlated with the safety at the same time. What's more, it shows that using different algorithm for mutual checking can hold system's safety even though each algorithm may be not so efficient. Profit evaluations of such hot standby and advices to design are also given.

Index Terms—reliability; safety; PUSH system design; soft hot standby

I. INTRODUCTION

Nowadays, in the impact of Internet, the digital TV industry in China is undergoing profound changes. In order to provide customers with high-quality video services, state-owned operators use technologies such as VOD, PUSH VOD, cloud video platform. Among them, PUSH VOD systems are serving millions of Chinese families. So the system's reliability and improvement have been urgent to be studied.

As we know, redundant standby systems are very wide applied, such as PLC systems [1], supply systems[2]. And from the view of types, redundant system can be classified into numerous ones, including standby redundant, parallel redundant and so on [3]. For k-n out warm standby, preventive maintenance, distribution systems reliability, lots of good study have been done[4]-[7].

According to whether there is a common storage medium, the hot standby can be divided into 2 ways: shared storage and pure-software styles. The former style is more reliable, the latter one costs less but need content synchronization. This paper studies reliability and safety of a soft hot standby in PUSH VOD system. Part II gives a short explanation for the system's redundant mechanism. Then Part III builds up an evaluation model for such system's reliability and safety, and gives a deep insight into some key parameters' roles and their relationship, which turns out to be some interesting rules for systems' design. Part IV provides a simple formula to judge the profit of standby. Finally, Part V makes a short conclusion and discusses some topics that need to be studied further.

II. MECHANISM OF SOFT HOT STANDBY PUSH VOD SYSTEM

The mechanism is shown in Fig. 1. We should notice that not only 2 PUSH server software works in a master-slave mode, but also there're 2 additional monitor software running in such mode. And we see this as a mutual checking standby network.

III. STUDY ON THE RELIABILITY OF SOFT HOT STANDBY PUSH VOD SYSTEM BASED ON MARKOV MODEL

A. Basic Hypothesis

Markov model is a portrait of a dynamic state transition matrix. In order to facilitate this model study, we need to make the following hypothesis .

Hypothesis 1: In the aspects of hardware and software, there is no different between master and slave machine.

Hypothesis 2: Master and slave software's failure rate is the exponential distribution and independent of each other.

Hypothesis 3: The errors in the system can be divided into the observable and the unobservable, technician will repair error in a reasonable time.
B. Basic Model of Reliability and Safety

We define the monitor software’s failure rate is $\lambda_0$, PUSH server’s is $\lambda_i$. Mutual detection ratio for monitors is $\kappa_n$. Mutual detection ratio for monitor and PUSH server is $\kappa_i$. To the same hot standby software failure, the PUSH server and monitor software simultaneously detection ratio is $\kappa_i$. Also, failure can be divided into the security failure and the hazard failure[8]. So a hazard ratio is defined as $\eta$. At the same time, we set up the technicians’ fixed expenses as $C_0$. Unit cost per up time is $C_1$. Success rate of repair is $\mu$ and system’s revenue per up time is $B_0$.

With these parameters, we define the following possible states:

State 1: All unit work properly, PUSH system can provide video push service.

State 2: When a machine is working properly and the other is to be isolated, the system is in a controlled state. There are several possible reasons to make this happen:

(1) Master PUSH software has an observable failure, the slave software startup[9].

(2) The opposite situation of the above one.

In both situations, the PUSH system is under control.

State 3: Either Master or slave has an unobservable failure. In this state, the system may operate normally, but is out of control. For example, the master machine may have some trouble, but due to the unobservable, master’s PUSH software will continue to run. The system is still normal to provide service for customers. However, when the master PUSH software fails, the standby will start up. At this point, the whole hot standby system state is undefined, and the system enters into a dangerous state.

State 4: System security failure state. In this state, the whole system will stop providing video output, but everything is under control. The reasons can cause this are as follows:

(1) When only one machine is at work, this working machine’s PUSH software happened to occur an observable error.

(2) When only one machine is at work, this working machine’s monitor software happened to occur an observable error.

State 5: System risk failure state. Reasons as below:

(1) When system start up, master has an observable error.

(2) When system is in State 2, an unobservable error happened.

(3) When system is in State 3, an unobservable error happened.

Fig. 2 shows the model of state transition. Then, we can get the following differential equations (1):

\[
P'_1(t) = -(2\lambda_0 + \lambda_i)P_1(t) + \mu P_2(t) + \mu P_3(t)
\]

\[
P'_2(t) = [\lambda_i \kappa_i + 2\lambda_0(\kappa_i - \kappa_n)]P_1(t) - (\lambda_i + \mu)P_2(t)
\]

\[
P'_3(t) = -\lambda_i P_1(t) + [2\lambda_0(1-\kappa_n - \kappa_i + \kappa_i)]P_1(t)
\]

\[
P'_4(t) = \lambda_i P_1(t) + [\lambda_i(1-\kappa_i) + \lambda_i \kappa_i \eta]P_2(t)
\]

\[
P'_5(t) = \lambda_i P_1(t) + \lambda_i \kappa_i \eta P_2(t) - \mu P_2(t)
\]

\[
P'_6(t) = (\lambda_i \kappa_i + 2\lambda_0(\kappa_i - \kappa_n))P_1(t) - (\lambda_i + \mu)P_2(t)
\]

In order to study the characteristics of this set of equations, we do a simple processing. Assuming no maintenance staff repairs machine and improve software, we set $\mu = 0$. Then, simplified equations can be solved with initial conditions $P(0)=[1 0 0 0 0]$:

\[
P_1(t) = e^{-(2\lambda_0 + \lambda_i)t}
\]

\[
P_2(t) = \frac{[\lambda_i \kappa_i + 2\lambda_0(\kappa_n + \kappa_i - \kappa_i)]}{\lambda_i + \lambda_i} e^{-(\lambda_i + \lambda_i)t}
\]

\[
P_3(t) = \frac{[2\lambda_0(1-\kappa_n - \kappa_i + \kappa_i)]}{\lambda_i + \lambda_i} e^{-(\lambda_i + \lambda_i)t}
\]

\[
P_4(t) = \frac{\lambda_i \kappa_i (1-\eta)[\lambda_i \kappa_i + 2\lambda_0(\kappa_n + \kappa_i - \kappa_i)]}{\lambda_i + \lambda_i}
\]

\[
\eta = \frac{1}{\lambda_0} e^{-\lambda_i t} + \frac{1}{2\lambda_0 + \lambda_i} e^{-(2\lambda_0 + \lambda_i)t}
\]

Set $H = \frac{\lambda_i}{\lambda_0}$, $K = \kappa_i + \kappa_i - \kappa_i$

Then, we define the reliability and safety of soft hot standby respectively:

\[
R(t) = P_1(t) + P_2(t) + P_3(t) = e^{-\lambda_i (2H + 1)t}
\]

\[
S(t) = R(t) + P_4(t)
\]

\[
= [1 + \frac{\kappa_i (1-\eta)(\kappa_i + 2HK)}{(2H + 1)(H + 1)}] e^{-\lambda_i (2H + 1)t}
\]

Not surprisingly, the reliability shows an exponential distribution. But it is worth to notice that the mutual checking ratio turns out to be a linear relationship with reliability. This is very strong positive factor to design a reliable system. To the safety formula, it is hard to see...
some obvious clues. But we can guess there're some relationship between \( K \) and safety.

C. The Analysis of Reliability and Safety

In purpose of deep insight into reliability and safety, we set a number of parameters to see what will happen.

First, we take the \( H \) ’s influence into consideration.

Let \( \lambda_0 =0.05, \quad \kappa_0 =0.9, \quad H = \{1, 0.8, 0.6, 0.4, 0.2\} \), we get Fig. 3. Obviously, \( H \) doesn't play a big role as it seems to be. But we can think it in another positive way, that is even when \( H \) is very low which means \( \lambda_0 \) is very high, the PUSH system can hold a steady reliability. Because \( \lambda_0 \) is very low, and all software are checking each other. So let's put this idea further and get rule 1.

Rule 1 (anti-Cask-principle): In a system with a mutual check reliable network, the high reliable component not the low one hold the basic reliable level.

Next, we study the parameter \( \kappa_1 \). Set \( \lambda_0 =0.05, \quad H =0.1, \quad \kappa_1 = \{1, 0.9, 0.5, 0.3, 0.1\} \), we get Fig. 4. In this picture, no surprise, we get a rule 2.

Rule 2: Mutual check algorithm for monitor and service is strongly positive correlative to system’s reliability.

Then, we have to try on the safety analysis. We want to see how \( H \) can affect the safety. Let \( \lambda_0 =0.05, \quad K =0.9, \quad \kappa_0 =0.9, \quad \eta =0.1, \quad H = \{1, 0.95, 0.9, 0.85, 0.8\} \), Fig. 5 jumps out in front of us. Surprisingly, \( H \) is strongly negative correlative with the safety. This result implied that when PUSH server shuttles down, this error can be guided into State 4 with high probability. So we also can get another rule.

Rule 3: In mutual check standby system, some internal parameter may cause paradoxes between reliability and safety. (\( H \) is such a case.)

Then, Let's study \( \kappa_1 \). Set \( \lambda_0 =0.05, \quad \kappa_0 =0.9, \quad \kappa_1 =1, \quad H =1, \quad \kappa_1 = \{0.95, 0.9, 0.8, 0.7, 0.5\} \), Fig. 6 shows up. Keeping consistency with reliability, \( \kappa_1 \) is strongly correlative with safety. This is rule 4.

Rule 4: Mutual check algorithm for monitor and service is strong positive correlative to system’s safety.

Then, because there’re 2 kinds of mutual checking, we want to know whether the diversity of checking algorithm
will make a contribution to the safety. So let \( \lambda_0 = 0.05, \kappa_1 = 0.5, \kappa_0 = 0.5, H = 0.4, \kappa_2 = [0.95 0.8 0.7 0.5 0.2] \), Fig. 7 demonstrates an exciting fact that even \( \kappa_0, \kappa_1 \) are pretty low, we can still get a relatively high safety if \( \kappa_0, \kappa_1 \) algorithm have enough different. That is:

Rule 5: Taking different algorithm for 2 types of mutual check will obviously improve safety.

Finally, we add maintenance staffs into consideration. We rewrite equations(1) in matrix and use Markov steady state analysis.

\[
\begin{align*}
P(t) &= 1 - 2\lambda_0 - 3\mu \\ P_i(t) &= \frac{\lambda_i(1 - \kappa_1 - \kappa_0 - \kappa_i) 0 - \lambda_1 0 0}{0 \lambda_1(1 - \eta) 0 - \mu 0} \\
P_j(t) &= \frac{\lambda_j(1 - \kappa_1 - \kappa_0 - \kappa_j) + \lambda_k \kappa_l \lambda_1 0 - \mu}{0}
\end{align*}
\]

(2)

**TABLE I. RELIABILITY AND SAFETY OF THE STEADY SYSTEM**

<table>
<thead>
<tr>
<th>Case</th>
<th>( R(\infty) )</th>
<th>( S(\infty) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.9467</td>
<td>0.9467</td>
</tr>
<tr>
<td>B</td>
<td>0.7279</td>
<td>0.6797</td>
</tr>
<tr>
<td>C</td>
<td>0.8142</td>
<td>0.8145</td>
</tr>
<tr>
<td>D</td>
<td>0.7052</td>
<td>0.6337</td>
</tr>
<tr>
<td>E</td>
<td>0.6910</td>
<td>0.4922</td>
</tr>
<tr>
<td>F</td>
<td>0.2647</td>
<td>0.4227</td>
</tr>
</tbody>
</table>

Let \( \lim_{t \to \infty} P_i(t) = \pi_i, j=1, 2, 3, 4, 5 \) and \( \sum_{j=1}^{5} \pi_j = 1 \).

When \( t \to \infty \), the PUSH system tend to be steady. With these conditions, we can solve (b). Because the result's algebraic expression is too complex, we do not list \( \pi_i \)'s expressions in the paper. Instead, we take some specific value to see what we can find in steady state analysis.

Let \( \nu = [\mu \lambda_0 \lambda_1 \kappa_0 \kappa_1 \kappa_2] \), then the above Table I shows some results.

A and B's data shows maintenance staffs' repair work improve system's reliability and safety. C's data again proves Rule 5. Looking at D and E's data, it shows that when PUSH server and monitor software failure rate drop dramatically, the reliability can hold a steady state but the safety will drop greatly. F's data means the system will be in a dangerous state if there is lack of mutual checking and maintenance staffs.

**IV. PROFIT ANALYSIS OF SOFT HOT STANDBY**

With parameters mentioned in part III, we define profit function as

\[
P(\pi) = \frac{B_0 \pi_1}{B_0 \pi_1 + C_0 (\pi_2 + \pi_3 + \pi_4) + C_0}
\]

**V. CONCLUSION**

Redundant standby systems have been widely applied in industry. With quantitative analysis of a soft hot standby system, we get some interesting rules. These rules can guide us to build a more reliable and safer system. But how to judge the difference level between mutual checking algorithm, how the topology structure of the reliable network influence systems' reliability and safety, these are questions that need further study.

**REFERENCES**


